Efficient Propagation of Imprecise Probabilities

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Abstract

It is often the case that sparse statistical data prohibits the assignment of a precise probability distribution to a given uncertain variable. In these cases, conventional statistical approaches such as Bayesian inference provide uncertain probability distributions - or imprecise probabilities. For convenience, many probabilistic approaches assign a specific distribution based on Maximum Likelihood Estimation (MLE) or some other criteria. Yet propagation of this single MLE distribution ignores potentially importance uncertainty/variability in these parameters. Meanwhile, some quasi-probabilistic approaches treat the uncertainties in the distribution using intervals (e.g. mean value of a distribution may be assigned an interval) and propagate a family of distributions to construct a probability-box (or p-box) to bound the probabilities, see Beer et al. (2013). This approach is generally computationally very expensive requiring several Monte Carlo analyses to propagate a large number of distributions.

In this work, we propose an approach based on importance sampling to propagate imprecise probability distributions with a single Monte Carlo analysis. The approach uses Bayesian inference to quantify the imprecise probabilities - determining a set of possible candidate distributions (that may come from different families - e.g. normal, lognormal, gamma, etc.) weighted according to their probability of occurrence. We then identify an optimal sampling distribution that shares the same support as the candidate distributions by solving an optimization problem to identify the coefficients of a Kernel density function that best represents all possible candidate distributions. Samples from this optimal Kernel density are propagated using Monte Carlo simulation and are re-weighted according to the different candidate distributions. Hence, we achieve the propagation of many probability distributions with a single Monte Carlo simulation. A further advantage of the methodology is that the underlying probability models can be updated using Bayesian updating and propagation of these updated distributions does not require additional simulations. Instead, the weights associated with the existing simulations are updated directly to update the probability model of the solution.

References

Beer, M., S. Ferson and V. Kreinovich. Imprecise probabilities in engineering analyses. *Mechanical Systems and Signal Processing*, 37(1-2):4–29, 2013.