Voting Aggregation Leads to (Interval) Median

O. Kosheleva[1)] and **V. Kreinovich**[2)]

[1)]Department of Teacher Education, University of Texas at El Paso, El Paso, TX 79968, USA
olgak@utep.edu

[2)]Department of Computer Science, University of Texas at El Paso, El Paso, TX 79968, USA
vladik@utep.edu

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Abstract

For many real-real problems, there are several different decision making tools. Each of these tools has its advantages and its limitations (otherwise, if a tool does not have any advantages, it would not be used). To combine the advantages of different tools, it therefore desirable to aggregate their results.

One of the most widely used methods of aggregating several results is voting: if the majority of results satisfy a certain property, then we conclude that the actual value has this property. For example, in a medical classification problem, if most classifiers classify the person's data as corresponding to pneumonia, we conclude that this person has pneumonia.

Of course, for the voting idea to work, we must limit ourselves to a reasonable class of properties – since, as one can easily show, no conclusion satisfies *all* majority-supported properties.

In this paper, we analyze what happens if we apply this idea to several estimates x_1, …, x_n of the same quantity. In this case, a reasonable class of properties consists of the properties $x \in [a, b]$ for different intervals $[a, b]$. With this choice, when we combine an odd number of estimates x_1, …, x_n with $n = 2k + 1$, voting results in selecting the median $x_{(k+1)}$. When we combine the even number of estimates $x_1, \ldots, x_n$, with $n = 2k$, we get an interval version of the median: the interval $[x_{(k)}, x_{(k+1)}]$.

This result holds if instead of voting, we consider the narrowest aggregation operation, which is invariant relative to arbitrary strictly increasing or strictly decreasing transformations.

A similar result holds, if we combine multi-D estimates $x_i = (x_{i1}, \ldots, x_{id})$. In this case, as reasonable properties, we can take the properties of belonging to a box $[\underline{a}_1, \overline{a}_1] \times \ldots \times [\underline{a}_d, \overline{a}_d]$, corresponding to all possible boxes. Then, the result of the voting aggregation is a box formed by interval medians.

Similar results hold, if instead of boxes, we allow all possible convex sets.