Why Min-Based Conditioning

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Abstract

In many practical situations, we do not have full information about which alternatives are possible and which are not. In such situations, an expert can estimate, for each alternative, the degree to which this alternative is possible. Sometimes, experts can produce numerical estimates of their degrees, but often, they can only provide us with qualitative estimates: they inform us which degrees are higher, but do not provide us with numerical values for these degrees.

After we get these degrees from the experts, we often gain additional information, because of which some alternatives, which were previously considered possible, are now excluded. To take this new information into account, we need to appropriately update the corresponding possibility degrees. In this paper, we prove that under several natural requirements on such an update procedure, there is only one procedure that satisfies all these requirements – namely, the min-based conditioning.

Definition 1. *Let* Ω *be a finite Universe of discourse. A* possibility distribution *if a function* $\pi : \Omega \to [0,1]$ *for which* $\max\limits_{\omega \in \Omega} \pi(\omega) = 1$.

Definition 2. *By a* conditioning operator*, we mean a mapping* $(\pi \mid \Psi)$ *that inputs a possibility distribution* π *on a set* Ω *and a non-empty set* $\Psi \subseteq \Omega$ *and returns a new possibility distribution for which* $(\pi \mid \Psi)(\omega) = 0$ *for all* $\omega \notin \Psi$.

C1. If $\pi_{|\Psi} = \pi'_{|\Psi}$, i.e., if $\pi(\omega) = \pi'(\omega)$ for all $\omega \in \Psi$, then $(\pi \mid \Psi) = (\pi \mid \Psi)$.

C2. If $\pi(\omega) < \pi(\omega')$ for some $\omega, \omega' \in \Psi$, then $(\pi \mid \Psi)(\omega) < (\pi \mid \Psi)(\omega'), \ldots$

Proposition. *The only conditioning operator that satisfies the properties* **C1–C6** *is the min-based operator for which* $(\pi \mid \Psi)(\omega) = 0$ *when* $\omega \notin \Psi$ *and:*

- $(\pi \mid \Psi)(\omega) = 1$ *when* $\omega \in \Omega$ *and* $\pi(\omega) = \max\limits_{\omega' \in \Omega} \pi(\omega')$*, and*

- $(\pi \mid \Psi)(\omega) = \pi(\omega)$ *when* $\omega \in \Omega$ *and* $\pi(\omega) < \max\limits_{\omega' \in \Omega} \pi(\omega')$.