Dealing with Vague and Limited Information in Uncertainty Quantification

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Thanks to:
Liam Comerford, Marco de Angelis, Zhang Mingqiang, …

GENERAL SITUATION

Endeavor

- numerical modeling – physical phenomena, structure, and environment

  ➡ prognosis – system behavior, hazards, safety, risk, robustness, economic and social impact, ...

» close to reality
» numerically efficient

Deterministic methods

- deterministic structural parameters
- deterministic computational models

⬌ Reality

Uncertainty ?

Imprecision ?

Vagueness ?

Fluctuations ?

Ambiguity ?

Variability ?

Indeterminacy ?

➡ Consequences ?

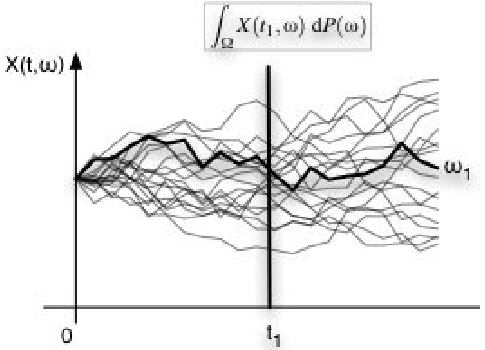
MULTI-DISCIPLINARY CHALLENGE

Major directions of development … and combinations thereof

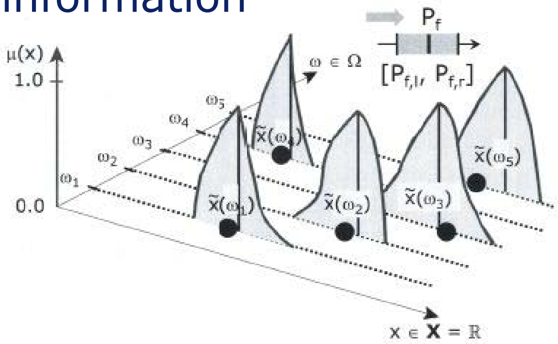
Advanced
Stochastic Modeling

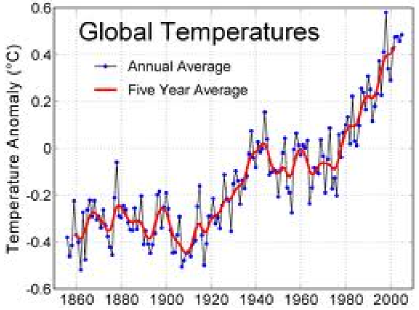
$$Z = \int Dx\, e^{iS[x]/\hbar}$$

Monte Carlo
Simulation methods



Generalized models
for vague and imprecise information

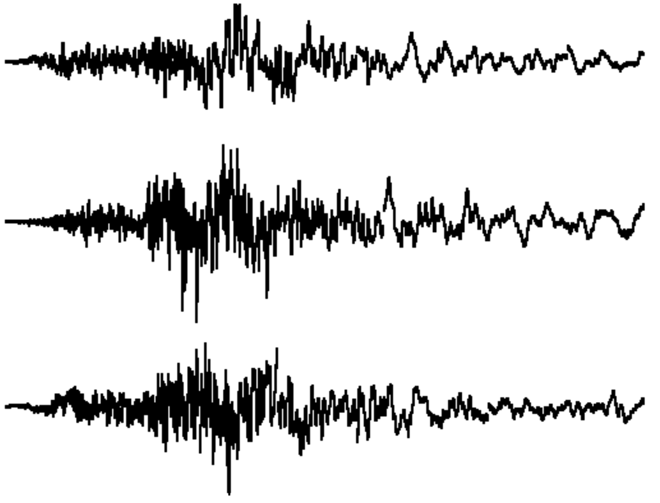




JOINT TIME-FREQUENCY ANALYSIS

Civil infrastructure

» seismic motions
» ocean waves
» winds
» blast events
» storms
» hurricanes



Extreme Events

JOINT TIME-FREQUENCY ANALYSIS

Excitation → System → Response

Excitation: non-stationary stochastic process

(earthquake, wind, ocean waves, blast events etc)

System: nonlinear and time-varying behavior due to severe dynamic excitation

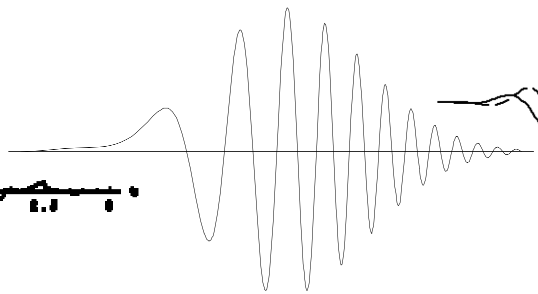
Response: non-stationary stochastic process

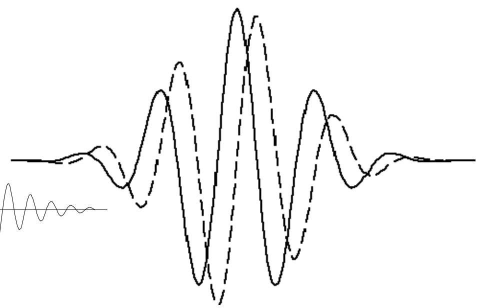
- excitations with time-varying intensity & frequency content
  → joint time-frequency analysis

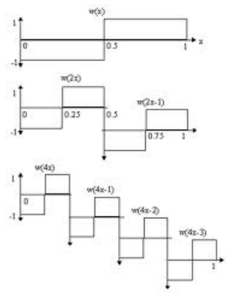
Wavelets

class of functions with compact support in both time and frequency domains used for the localization of a function in these domains

Alfred Haar (1909)





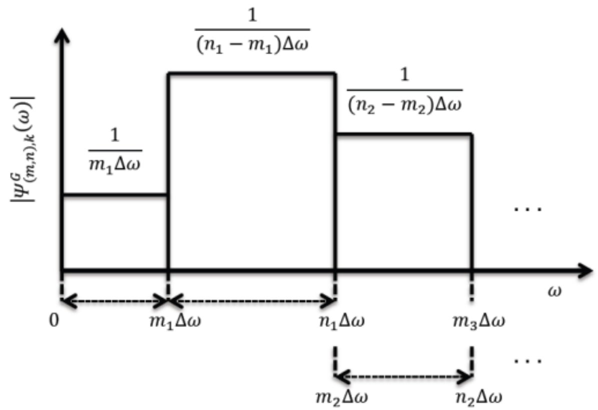


JOINT TIME-FREQUENCY ANALYSIS

Evolutionary Power Spectrum (EPS) estimation

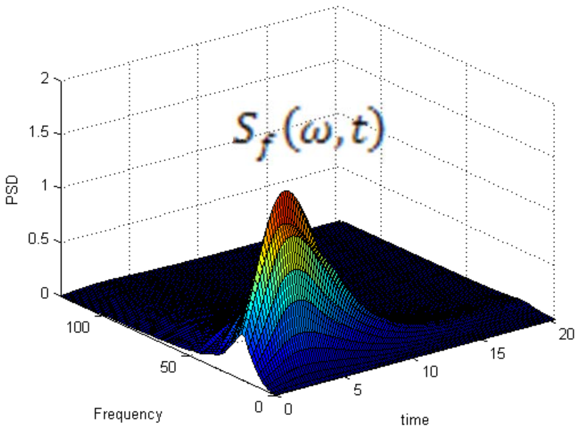
- mathematical model for random process representation
  - » representation of random properties in correspondence with the EPS
  - » locally stationary wavelet process

$$f(t) = \sum_j \sum_k w_{j,k} \psi_{j,k}(t) \xi_{j,k}$$



- Generalized Harmonic Wavelets
  - » orthogonality & additional parameter

➡ convenient frequency windowing
  - » flexible window size
  - » non-overlapping supports



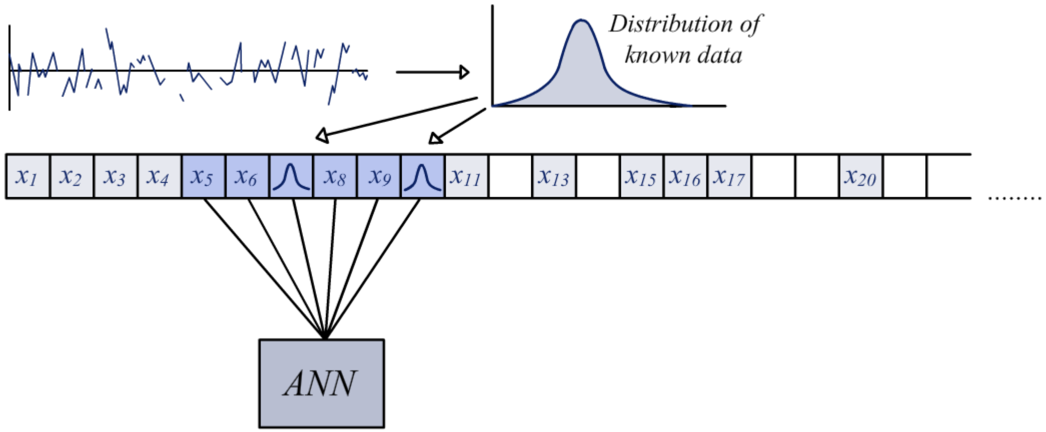
➡ distribution of “power” or “energy” of the underlying non-stationary stochastic process over different frequencies

basis for most engineering load models

JOINT TIME-FREQUENCY ANALYSIS

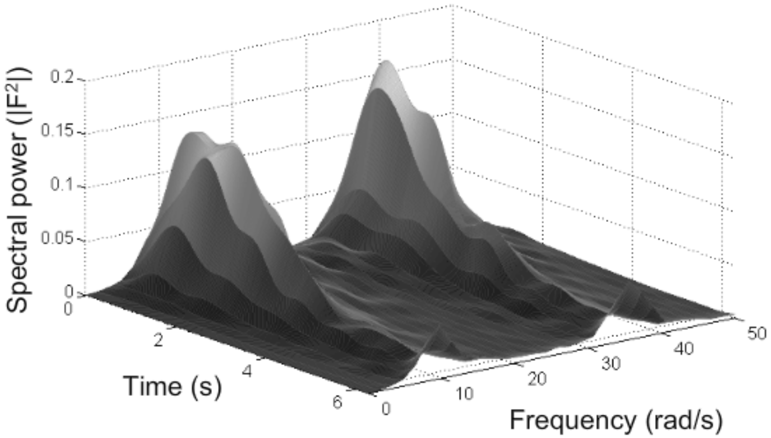
Incomplete data

- process records with gaps
  - » sensor failures
  - » data corruption
  - » power outages
  - » ...
- artificial intelligence technology



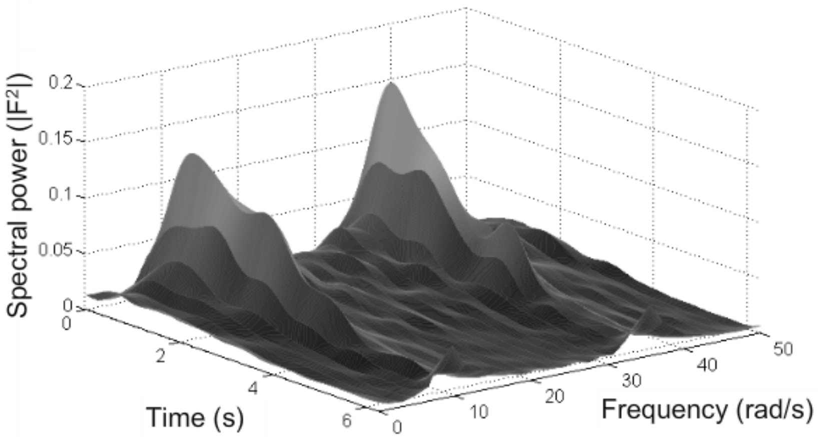
Harmonic wavelet estimated spectrum

25 records of length 512 - **no missing data**



Estimated spectrum of ANN reconstructions

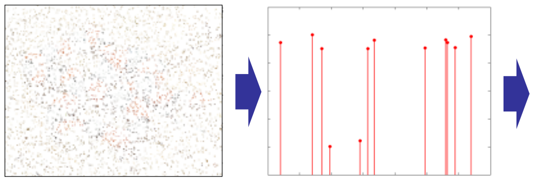
25 records of length 512 - **50% missing data in random locations**



JOINT TIME-FREQUENCY ANALYSIS

Incomplete data

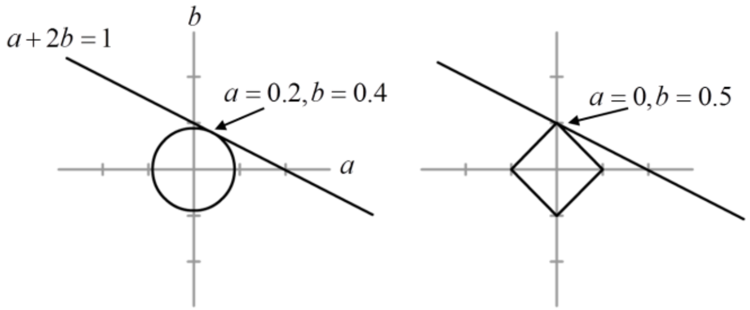
- Compressive sensing approach



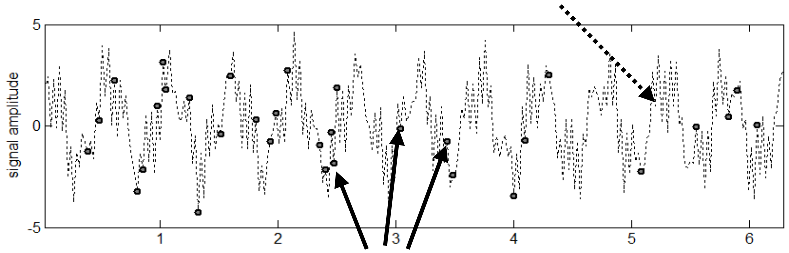
Assume sparsity in a known basis

- Minimization of $\Sigma_i|x_i|$ in $Ax = y$ promotes sparse solutions

Least squares **L1 minimization**

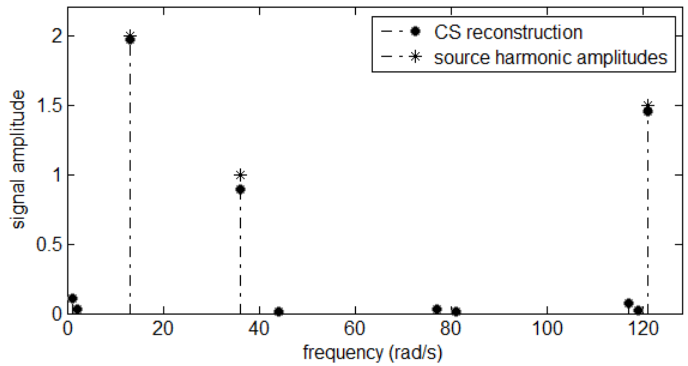


Stochastic process with 3 harmonics and white noise



Limited number of sample points available

Assume sparsity in harmonic basis to identify key frequencies

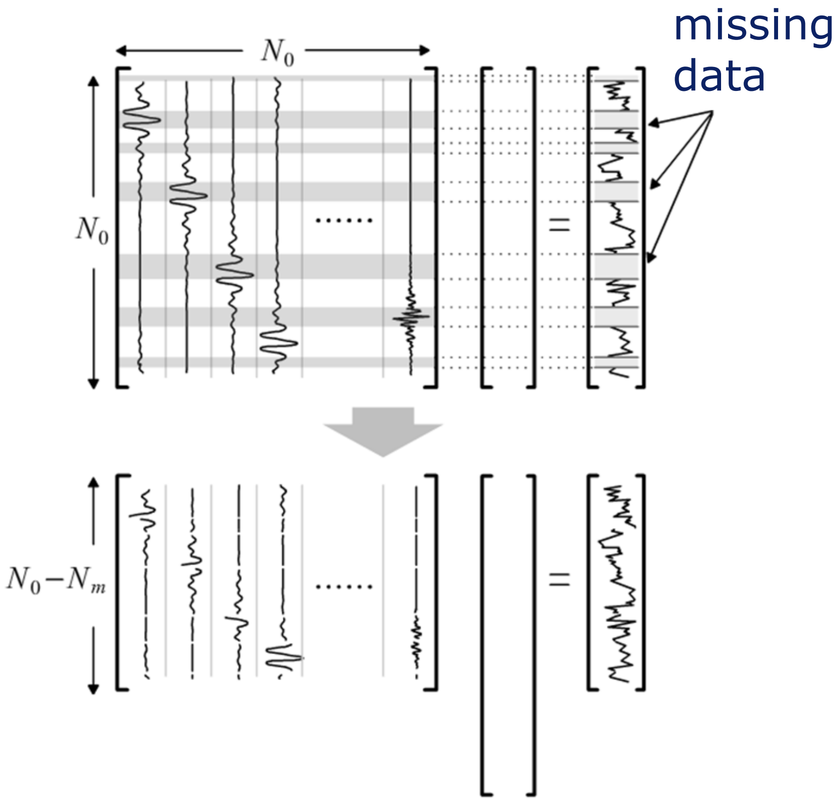


JOINT TIME-FREQUENCY ANALYSIS

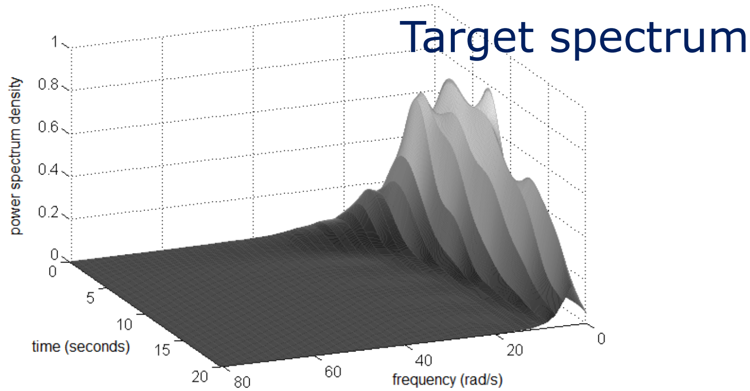
Incomplete data

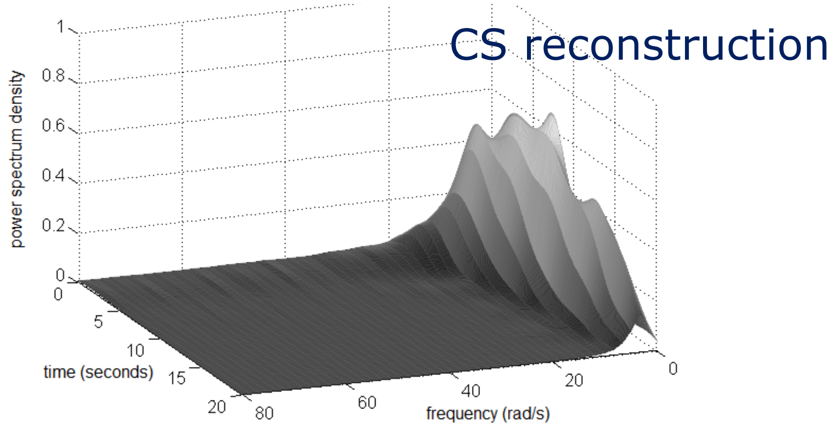
- Compressive sensing approach
  - » Harmonic basis construction

Wavelet basis (example)



Example for non-separable, non-stationary process: (50% randomly distributed missing data over 25 records using CS with a harmonic wavelet basis)

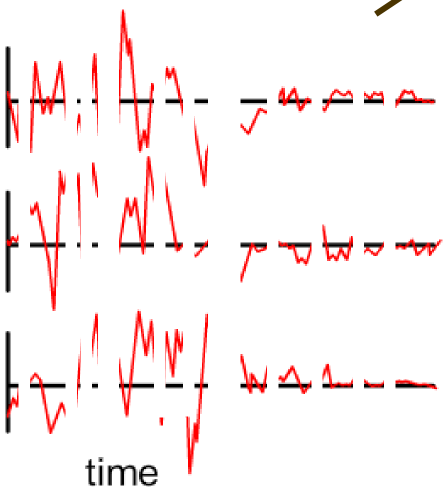




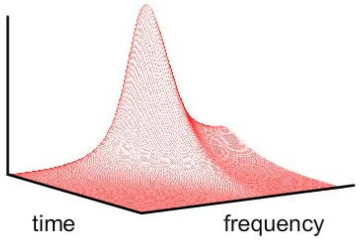
EXAMPLE: RELIABILITY ANALYSIS

Incomplete Earthquake Records

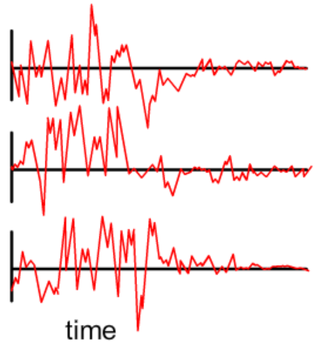
Incomplete
non-stationary
earthquake
records



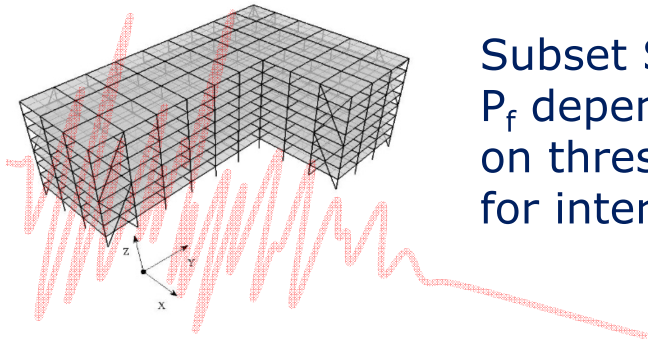
Spectrum estimation



Simulated
realizations



Reliability Analysis



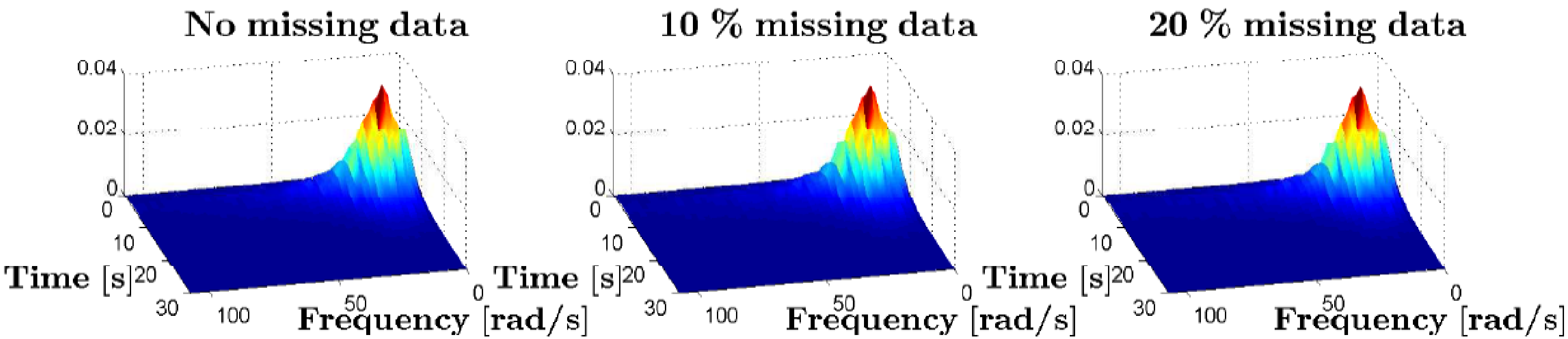
Subset Sampling;
P_f depending
on threshold
for interstory drift

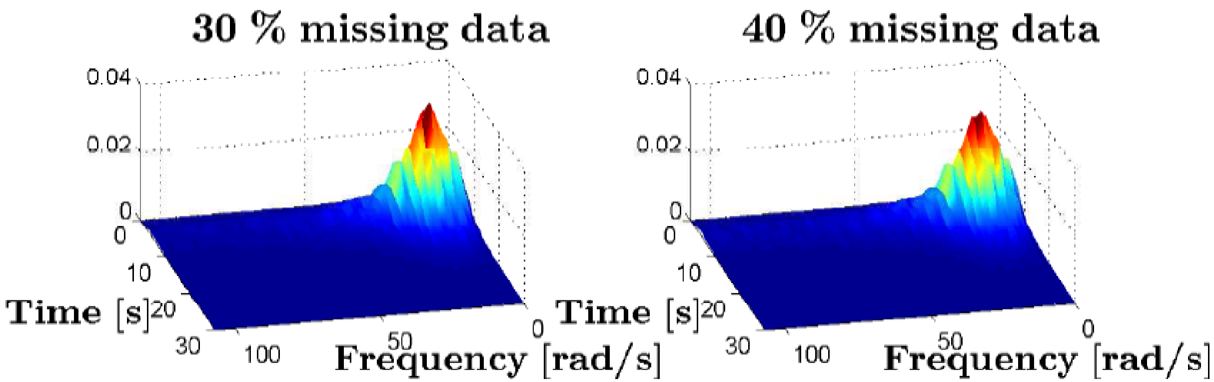
Collaboration with
Professor Hector Jensen

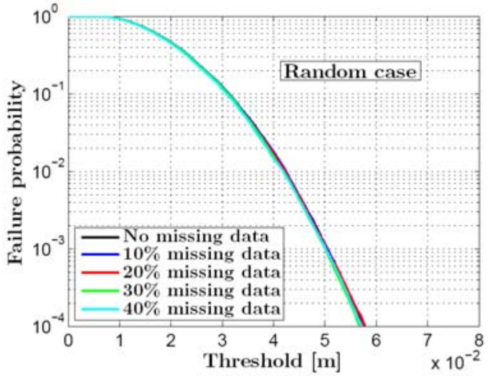
EXAMPLE: RELIABILITY ANALYSIS

Incomplete Earthquake Records

- Scenario 1: data removed at random positions (10%, 20%, 30%, 40%)







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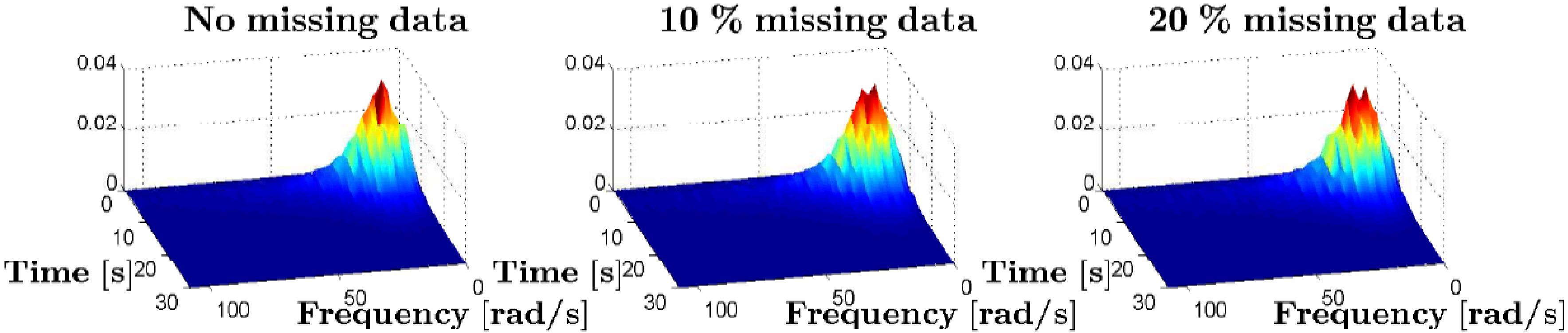
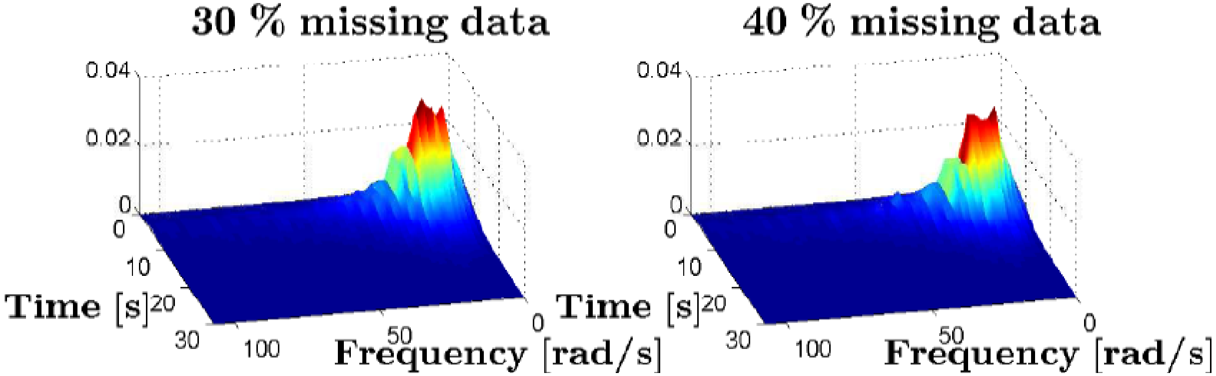
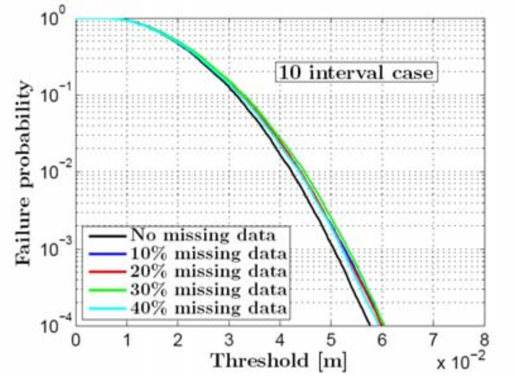
EXAMPLE: RELIABILITY ANALYSIS

Incomplete Earthquake Records

- Scenario 2: data removed at 10 intervals located at random positions (10%, 20%, 30%, 40%)







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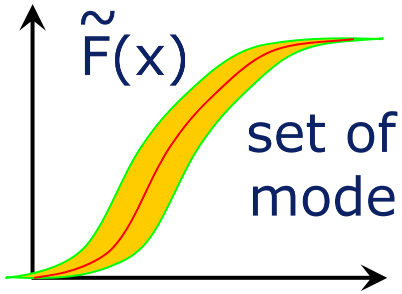
CHALLENGE

statistical analysis of imprecise and rare data

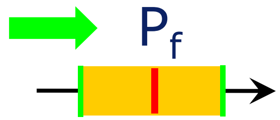
Is it safe ?



set of plausible models

reliability analysis

$\tilde{P}_f$



$[P_{f,l}, P_{f,r}]$
imprecision reflected in P_f

Is the reliability analysis still reliable ?

Effects on P_f ?

epistemic uncertainty

Sensitivity of P_f to imprecision ?

CLASSIFICATION AND MODELING

According to sources

- aleatory uncertainty
  - » irreducible uncertainty
  - » property of the system
  - » fluctuations / variability

  stochastic characteristics

  ➡ traditional probabilistic models

- epistemic uncertainty
  - » reducible uncertainty
  - » property of the analyst
  - » lack of knowledge or perception

  collection of all problematic cases, inconsistency of information

  ➡ no specific model

According to information content

- uncertainty
  - » probabilistic information

  ➡ traditional and subjective probabilistic models

- imprecision
  - » non-probabilistic characteristics

  ➡ set-theoretical models

In view of the purpose of the analysis

- averaged results, value ranges, worst case, etc. ?

CLASSIFICATION AND MODELING

Simultaneous appearance of uncertainty and imprecision

- separate treatment of uncertainty and imprecision in one model
- generalized models combining probabilistics and set theory

➡ **concepts of imprecise probabilities**

» interval probabilities
» sets of probabilities / p-box approach
» random sets
» fuzzy random variables / fuzzy probabilities
» evidence theory / Dempster-Shafer theory

common basic feature:

set of plausible probabilistic models over a range of imprecision

(set of models which agree with the observations)

➡ bounds on probabilities for events of interest

IMPLEMENTING EPISTEMIC UNCERTAINTY

Subjective probabilities

- Bayesian update

$$f_{\Theta|X_1,\ldots,X_n}\left(\theta|x_1,\ldots,x_n\right)=\frac{\left[\prod_{i=1}^{n} f\left(x_i|\theta\right)\right]\cdot g_{\Theta}\left(\theta\right)}{\int\left[\prod_{i=1}^{n} f\left(x_i|\theta\right)\right]\cdot g_{\Theta}\left(\theta\right)d\theta}$$

prior distribution

posterior distribution

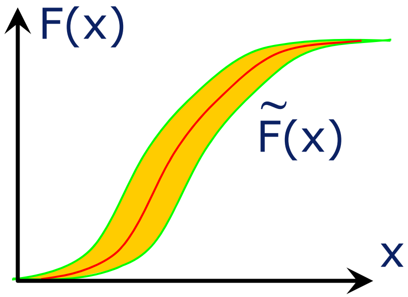
» approach result from "inside the epistemic uncertainty"

Imprecise probabilities

- set-theoretical models for indeterminacy / imprecision (imprecise data, vague conditions or copulas etc.)

$$F(x)=\left\{\left[F_l(x_i),F_u(x_i)\right]\ \forall x_i\right\}$$

e.g., set-valued parameters in probabilistic models



» approach result from "outside the epistemic uncertainty"

➡ choice depending on available information and purpose of the analysis

➡ approaches not competing but complementary and can be combined (e.g. set of priors, update with imprecise data)

FUZZY PROBABILITIES

Model

- fuzzy random variables $\tilde{X}: \Omega \rightarrow \mathbb{F}(\mathbb{R})$ (frequentist view)

  ➡ fuzzy set of real-valued random variables $\tilde{X} = \left\{ \left(X_j,\ \mu\left(X_j\right) \right) \ \middle|\ x_{ji} \in \tilde{x}_i \ \forall i \right\}$

  ➡ fuzzy set of cdfs $\tilde{F}(x) = \left\{ \left(F_j(x),\ \mu\left(F_j(x)\right) \right) \ \middle|\ X_{ji} \in \tilde{X},\ \mu\left(F_j(x)\right) = \mu\left(X_j\right) \ \forall j \right\}$

Relationship to other concepts of imprecise probabilities

- evidence theory
  - » focal subsets represented by fuzzy sets
  - » frequentist view: probability of focal subsets induced by elementary events
- p-box approach
  - » fuzzy set of p-boxes $\tilde{F}(x) = \left\{ \left(F_\alpha(x),\ \mu\left(F_\alpha(x)\right) \right) \ \middle|\ F_\alpha(x) = \left[F_{\alpha l}(x), F_{\alpha u}(x) \right],\ \mu\left(F_\alpha(x)\right) = \alpha \ \forall \alpha \in (0,1] \right\}$

Numerical processing

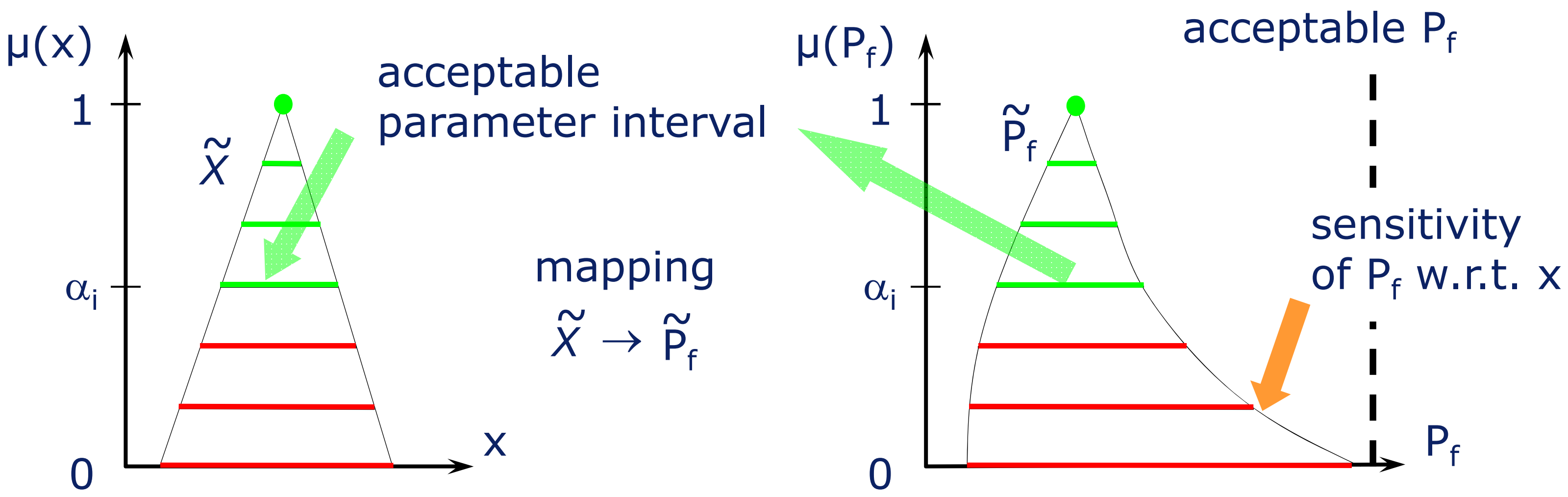
- stochastic techniques combined with interval / fuzzy analysis
  - » input: fuzzy set of plausible probabilistic models
  - » output: bounds on probabilities/probabilistic parameters for events/quantities of interest for several intensities of imprecision

FUZZY PROBABILITIES IN RELIABILITY ANALYSIS

Fuzzy parameters

- structural parameters
- probabilistic model parameters

Failure probability



- ➡ coarse specifications of design parameters & probabilistic models
- ➡ attention to / exclude model options leading to large imprecision of P_f
- ➡ acceptable imprecision of parameters & probabilistic models
- ➡ indications to collect additional information
- ➡ definition of quality requirements
- ➡ robust design

μ(.) not important, but analysis with various intensities of imprecision

EXAMPLE: RELIABILITY ANALYSIS

Fixed jacket platform

- imprecision in the models for
  - » wave, drag and ice loads
  - » wind load
  - » corrosion
  - » joints of tubular members
  - » foundation
  - » possible damage

Collaboration with Profs.
Quek Ser Tong
Choo Yoo Sang

EXAMPLE: RELIABILITY ANALYSIS

Collaboration with Profs.
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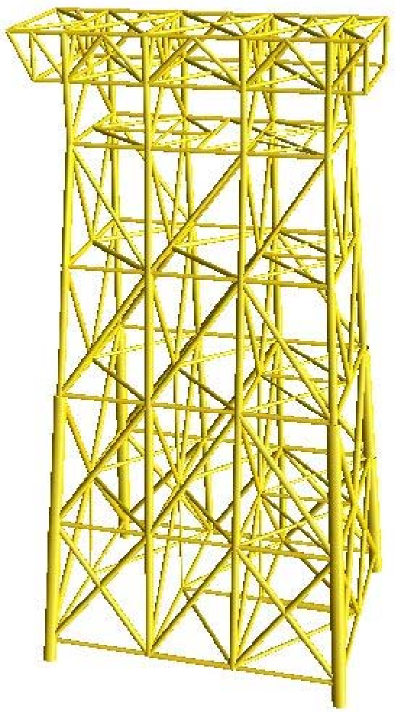
Choo Yoo Sang

Fixed jacket platform

- dimensions
  - » height: 142 m
  - » top: 27 X 54 m
  - » bottom: 56 X 70 m

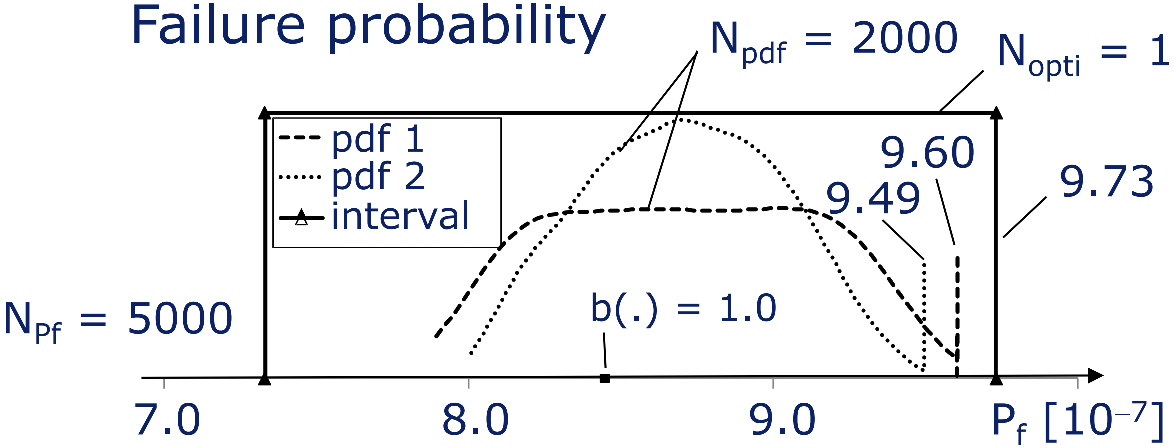


- loads, environment
  - » T = 15°C, t = 5 a
  - » random: wave height, current, yield stress, and corrosion depth c(t,E)
  - » pdf / interval for imprecise parameters

Reliability analysis

- Monte Carlo simulation with importance sampling and response surface approximation

Failure probability



STOCHASTIC SAMPLING

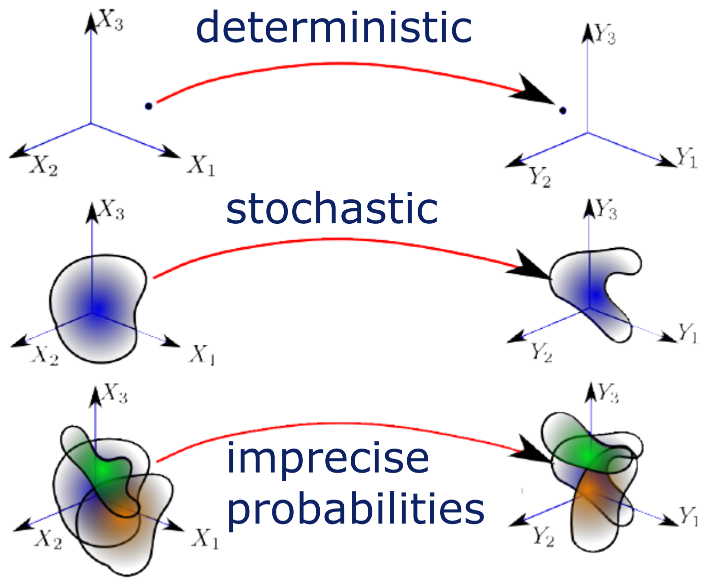
Features

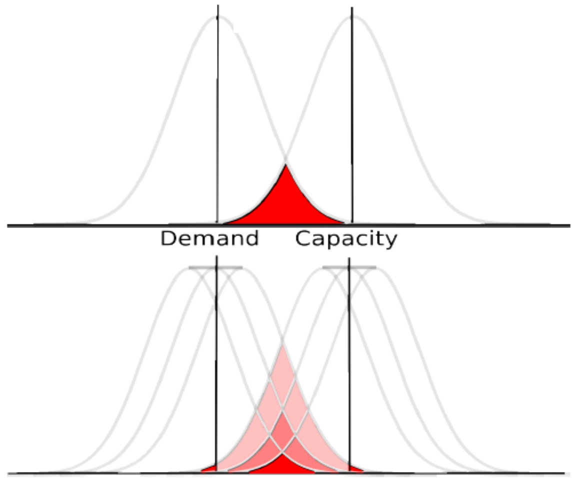
- generally applicable
  - » repeated analysis of deterministic model
  - » statistical evaluation of model response
- high numerical effort

➡ implementations to increase numerical efficiency
  - » high performance computing
  - » efficient (variance reducing) sampling schemes

➡ developments to further advance
  - » importance sampling
  - » subset sampling
  - » line sampling

in combination with subjective and imprecise probabilities

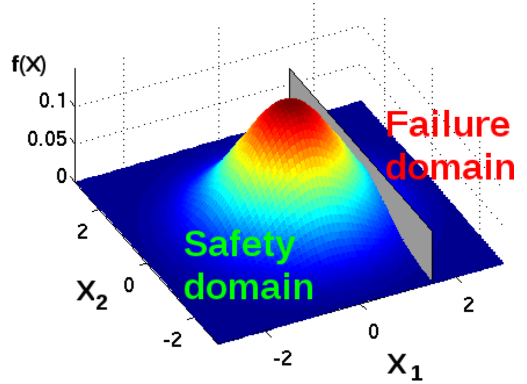




ADVANCED SAMPLING SCHEMES

Line sampling

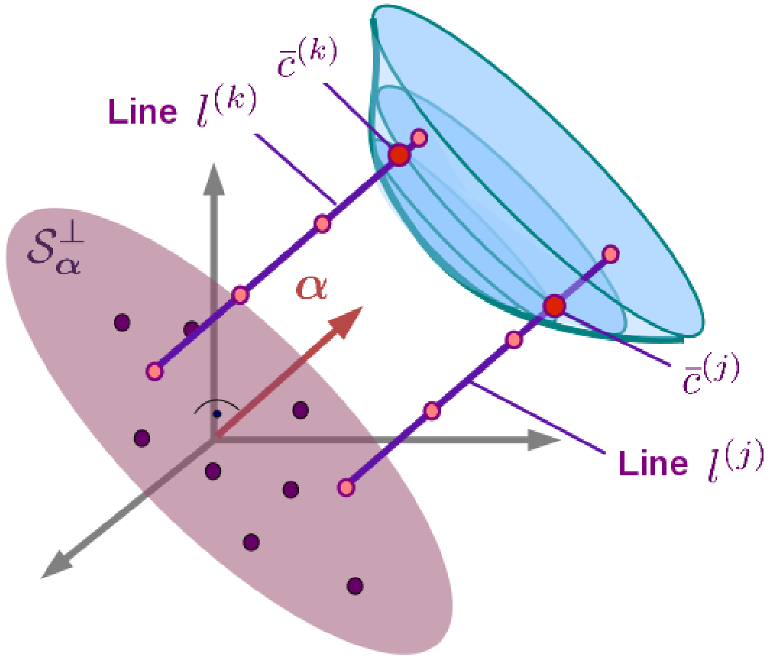
- concept
  - » in the standard normal space approximate P_f through distances to the limit state at random locations



- practical implementation
  - » build a subspace $S_\alpha^\perp$ perpendicular to the „important direction" α
  - » sample in $S_\alpha^\perp$ and find the distance $c^{(i)}$ to the limit state at the random points
  - » estimate P_f as average of the $p_f^{(i)}$ associated with the $c^{(i)}$

$$\hat{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} p_f^{(i)} \qquad \hat{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} \phi\left(-\bar{c}^{(i)}\right)$$

  - » independent of
    - dimensionality and
    - magnitude of P_f

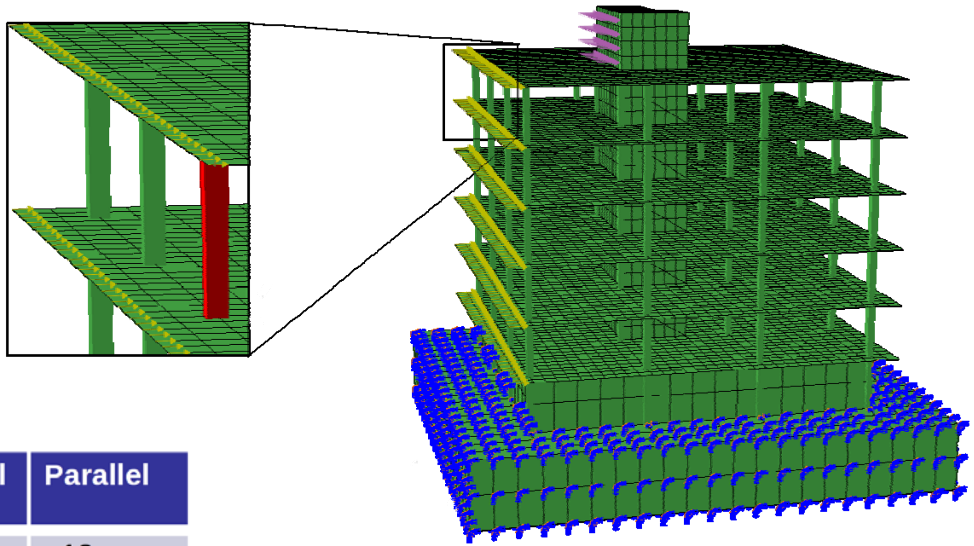


EXAMPLE: RELIABILITY ANALYSIS

Multi-storey building

- model
  - » 8,200 finite elements, 66,300 dof
  - » 244 random variables and 5 intervals (parameters of some rv)

- reliability analysis
  - » component failure
  - » line sampling
  - » interval analysis with global optimization
  - » distributed computing



Analysis type	Sequential	Parallel
Direct Monte Carlo Simulation 10000 samples	> 6 days	~13 hours
Advanced MCS – Line Sampling 100 samples	1h 32 min	7.8 min

Method	Speed-up
Distributed computing	11.7
Advanced Simulation	~ 100
Advanced Simulation + Distributed computing	> 1000

HIGH-DIMENSIONAL PROBLEMS

Advanced Monte Carlo simulation with interval probabilities

- global optimization problem

$$\underline{p_f} = \inf_{x,p} \int_{\Omega_f(x)} h_d(\xi, p)\, d\Omega$$

$$\overline{p_f} = \sup_{x,p} \int_{\Omega_f(x)} h_d(\xi, p)\, d\Omega$$

p – distribution parameters
ξ – random variables
x – intervals

Ω_f depends on intervals x !

- map intervals x to augmented probability space

$$\Omega \times X \to \Theta : \quad x \to \eta \in \mathbb{C}_x = \left\{ h_n\left(\eta; \underline{\overline{\mu_x}}, \sigma_x\right) \middle| \underline{\overline{\mu_x}} = \underline{\overline{x}} \right\}$$

- exploit topological properties of Θ for line sampling

sampling direction $-\nabla g$

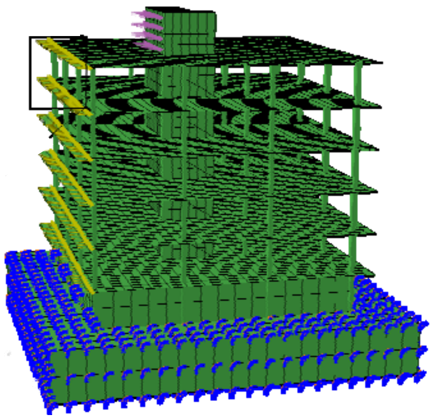
➡ optimal points $\left(p^u, x^u\right) = \psi^u\left(-\nabla g\right), \quad \left(p^l, x^l\right) = \psi^l\left(-\nabla g\right)$

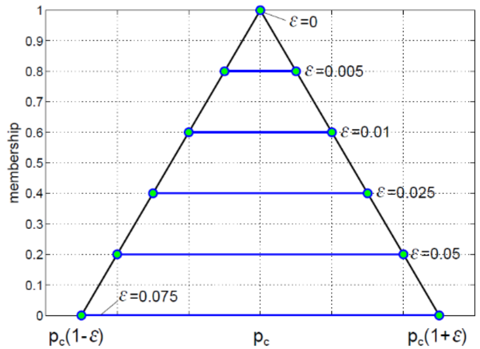
➡ $\overline{p_f} = \int_{\Omega_f(x^u)} h_d\left(\xi, p^u\right) d\Omega, \qquad \underline{p_f} = \int_{\Omega_f(x^l)} h_d\left(\xi, p^l\right) d\Omega$

EXAMPLE: ROBUST RELIABILITY ASSESSMENT

Multi-storey building – component failure analysis

- structure
  - » 8,200 finite elements, 66,300 dof
- imprecise probabilistic input
  - » 488 fuzzy parameters for 244 fuzzy random variables





± 7.5 %
tolerance range

SV #	Prob. dist.	$\overline{p} = p_c\ [1-\epsilon,\ 1+\epsilon]$		Description	Units
1	$\mathrm{N}(\overline{\mu},\ \overline{\sigma})$	$\mu_c = 0.1$	$\sigma_c = 0.01$	Columns' strength	GPa
2 – 193	$\mathrm{Unif}(\overline{a}, \overline{b})$	$a_c = 0.36$	$b_c = 0.44$	Sections' size	m
194 – 212	$\mathrm{LN}(\overline{m},\ \overline{v})$	$m_c = 35$	$v_c = 12.25$	Young's modulus	GPa
213 – 231	$\mathrm{LN}(\overline{m},\ \overline{v})$	$m_c = 2.5$	$v_c = 6.25\ 10^{-2}$	Material's density	$\mathrm{kg/dm}^3$
232 – 244	$\mathrm{LN}(\overline{m},\ \overline{v})$	$m_c = 0.25$	$v_c = 6.25\ 10^{-4}$	Poisson's ratio	-

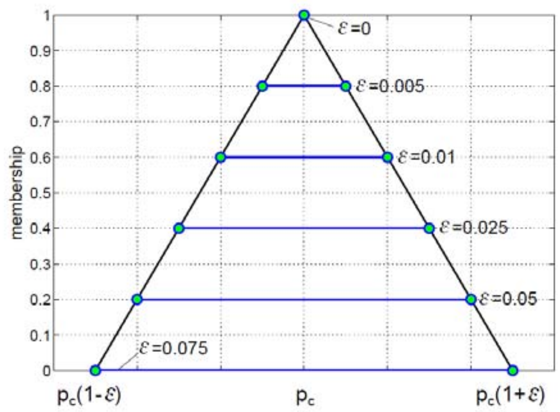
EXAMPLE: ROBUST RELIABILITY ASSESSMENT

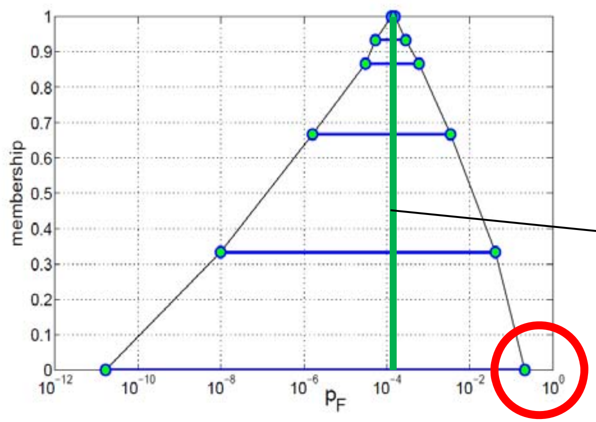
Multi-storey building – results

- advanced line sampling with pre-identified optimal points in Θ

	Lower Bound		Upper Bound		
ϵ	$\underline{p}_F$	CoV	$\overline{p}_F$	CoV	Ns
0.000	$1.42\ 10^{-4}$	$9.2\ 10^{-2}$	$1.42\ 10^{-4}$	$9.2\ 10^{-2}$	126
0.005	$5.75\ 10^{-5}$	$8.7\ 10^{-2}$	$2.63\ 10^{-4}$	$7.1\ 10^{-2}$	257
0.010	$4.57\ 10^{-5}$	$33.6\ 10^{-2}$	$5.30\ 10^{-4}$	$11.5\ 10^{-2}$	250
0.025	$1.75\ 10^{-6}$	$8.8\ 10^{-2}$	$3.22\ 10^{-3}$	$5.3\ 10^{-2}$	253
0.050	$2.27\ 10^{-8}$	$57.0\ 10^{-2}$	$3.88\ 10^{-2}$	$5.4\ 10^{-2}$	255
0.075	$1.88\ 10^{-11}$	$12.2\ 10^{-2}$	$2.02\ 10^{-1}$	$3.5\ 10^{-2}$	254

sample size
1,395





sensitivity of
failure probability

$P_f \approx 10^{-4}$

safety alerts

$\approx 10^{-1}$!

RESUMÉ

Major directions of development

$$\underbrace{\begin{array}{l}\bullet\ \text{advanced stochastic modeling}\\ \bullet\ \text{generalized models for vague and imprecise information}\\ \bullet\ \text{Monte Carlo simulation methods}\end{array}}_{\text{combinations}}$$

➡ realistic models

➡ efficient numerical analysis

➡ UQ for industry-sized structures and systems

➡ improved design, performance and reliability