TECHNISCHE UNIVERSITÄT DRESDEN

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Institute for Structural Analysis

A fail-safe design approach based on the fracture mechanical analysis and epistemic uncertainty quantification

A. Serafinska, K. Özenç, W. Graf, M. Kaliske

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Content

Failure due to catastrophic crack growth

origin

- modified loading, changed exploitation conditions
- unpredictable loading, e.g. impact loading
- fatigue of material
- usage errors

⇓

uncertainty

failure modelling with uncertainty analysis

Capturing failure in structural design

concepts

fail-safe

- safe failure
- minimal damage
- higher risk level accepted
- statically indeterminate
- efficient dimensioning

safe-life

- no failure
- predefined service life
- low risk level accepted
- significant structures
- statically determinate
- over dimensioned

damage-tolerance

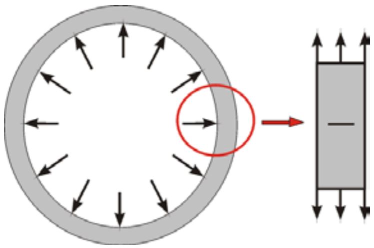
- predefined damage accepted
- damage till maintenance
- higher risk level accepted
- damage analysis
- reliable damage detection

Fail-safe design methods

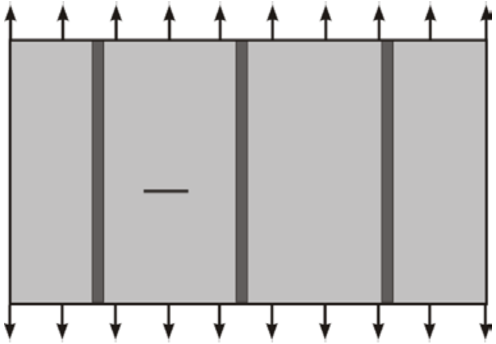
- redundancy of structural parts

- multiple load paths
  statically indeterminate

- early failure detection:
  - *monitoring, sensors*
  - *leak before break concept*

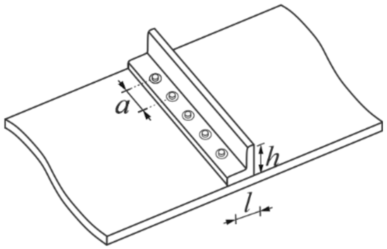


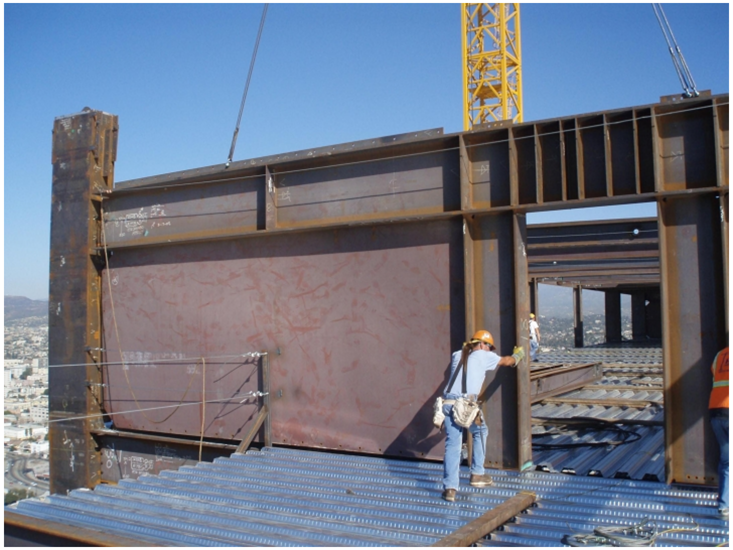
- limit crack propagation
  crack arrester, crack absorber

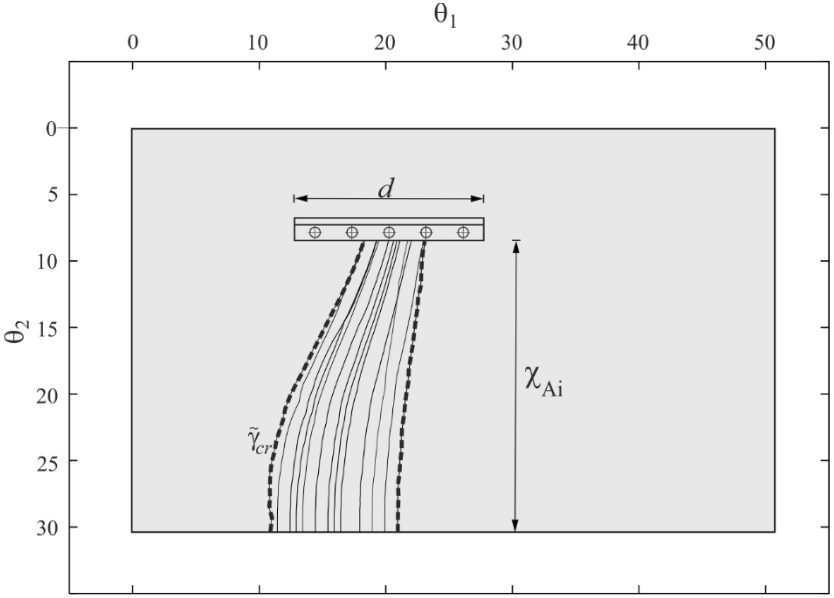


Fail-safe design problem

- unknown/uncertain crack initiation
- uncertain crack propagation
- crack propagation limiting elements



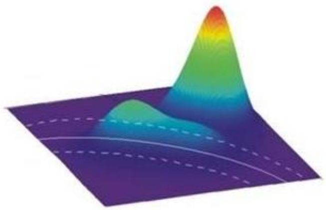




Coupled approach for fail-safe design

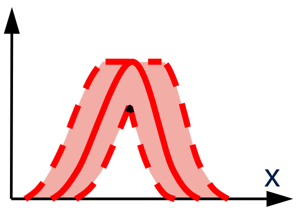
optimization $F_{OPT}(F_{UA}(M))$

binary genetic algorithm



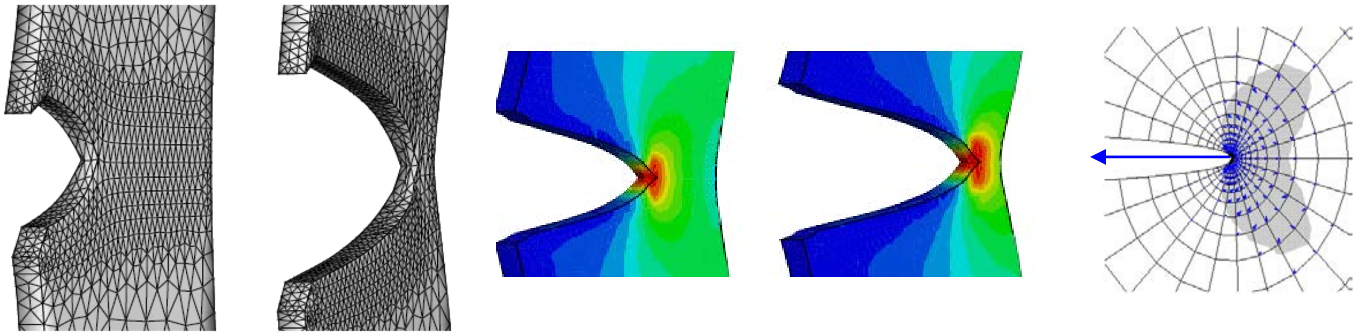
uncertainty analysis $F_{UA}(M)$

polymorphic uncertainty: fuzzy, probabilistic, fuzzy-stochastic



fracture mechanical solution M

finite element method
material force approach



Crack initiation as uncertain variable

crack initiation

↓

random nature?

↓

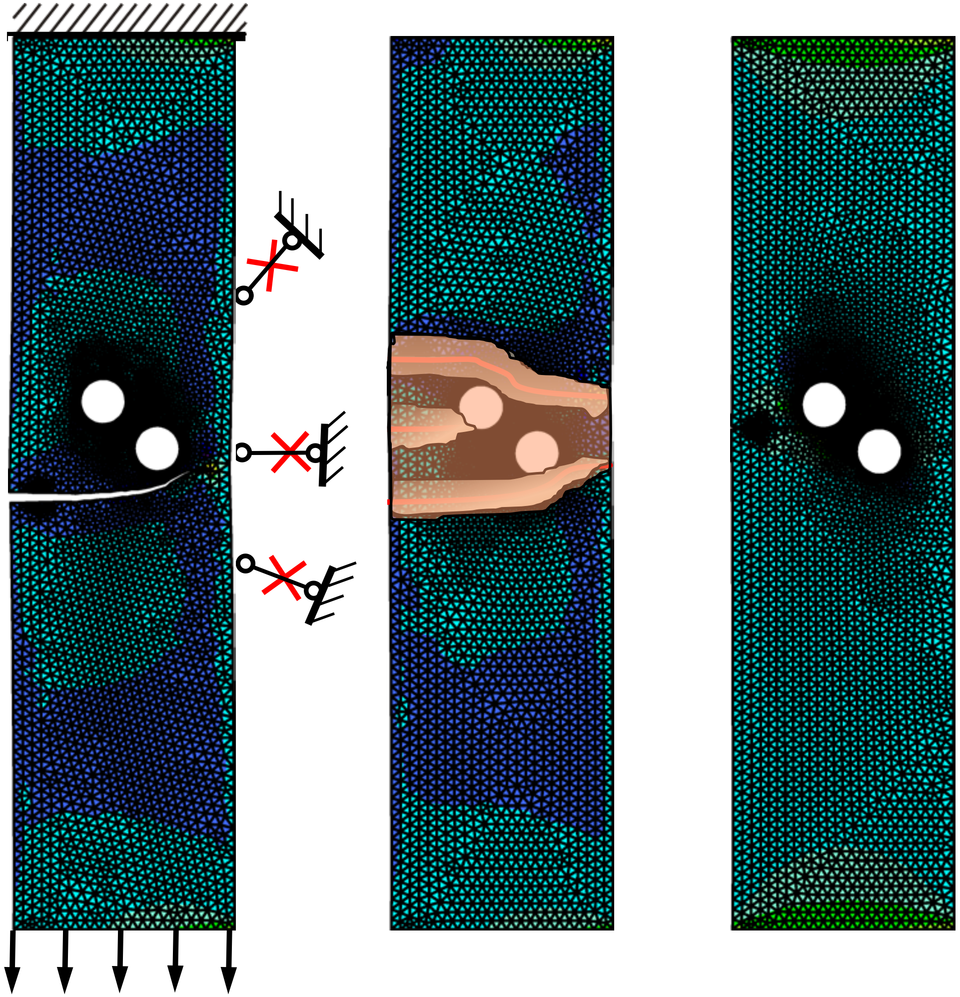
structural element in a system

↓

crack location as result of boundary change

↓

crack initiation location not purely random

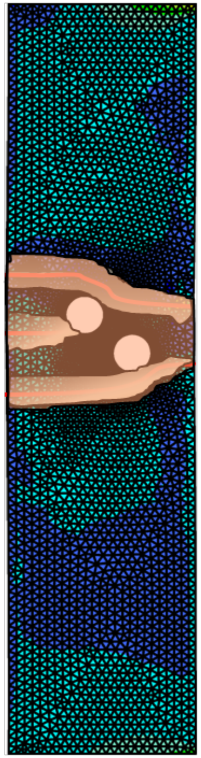


Failure due to boundary change

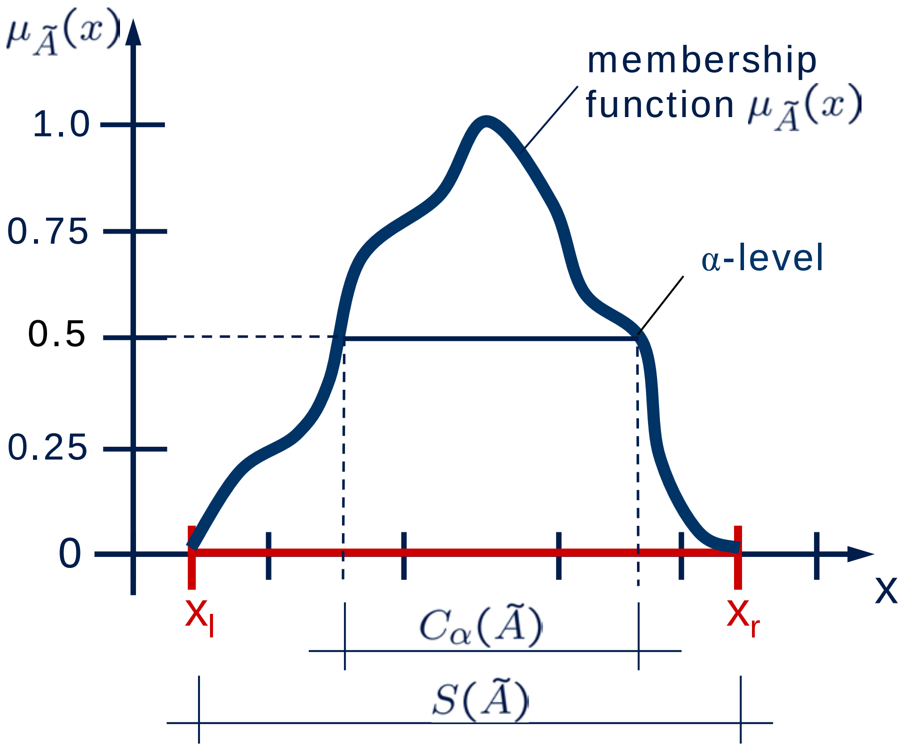
not purely stochastic character

↓

polymorphic uncertainty models applicable



Uncertainty model fuzziness



fuzzy set/fuzzy variable

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \mid x \in X \right\}$$

$$\mu_{\tilde{A}} : X \longrightarrow [0, 1]$$

$$S(\tilde{A}) = \{x \in X, \mu_{\tilde{A}}(x) > 0\}$$

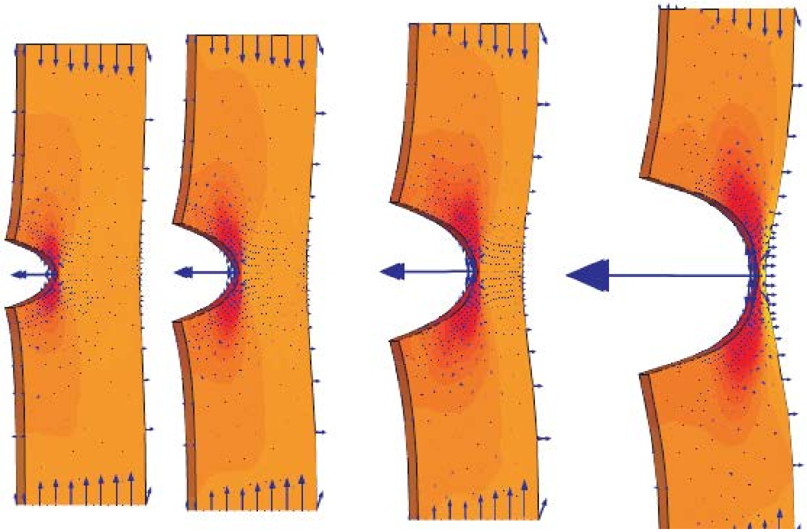
α –level discretization: $C_\alpha(\tilde{A}) = \{x \in X : \mu_{\tilde{A}} \geqslant \alpha\}$

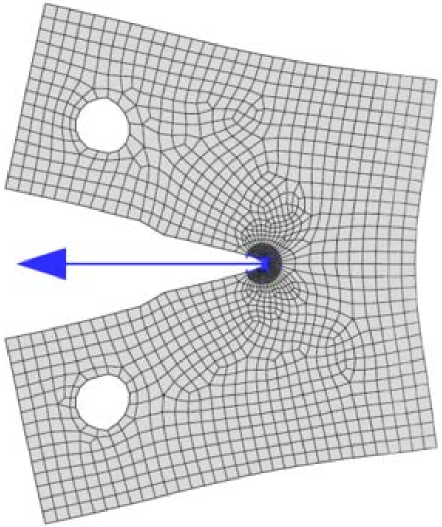
$$\tilde{A} = (C_\alpha(\tilde{A}))_{\alpha \in (0,1]}$$

convexity: $\mu_{\tilde{A}}(\lambda x_2 + (1 - \lambda)x_1) \geqslant min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$

Fracture mechanical model

- discrete fracturing
- crack propagation based on energy minimization principle
- r-adaptive node duplication
- configurational mechanics based fracture criteria
- configurational forces – crack driving forces





Computation of material forces

- local spatial momentum balance

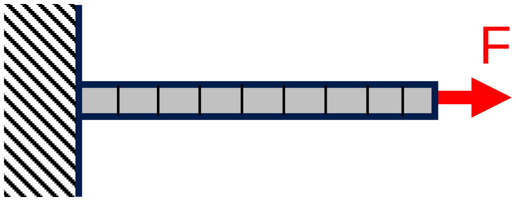
$$\nabla_X \cdot \mathbf{P}^T + \mathbf{b} = 0$$

$\mathbf{P} = \partial_{\mathbf{F}} \psi$

$\mathbf{P}$ first Piola-Kirchhoff stress

$\mathbf{b}$ body forces

ψ strain energy density



- local material momentum balance

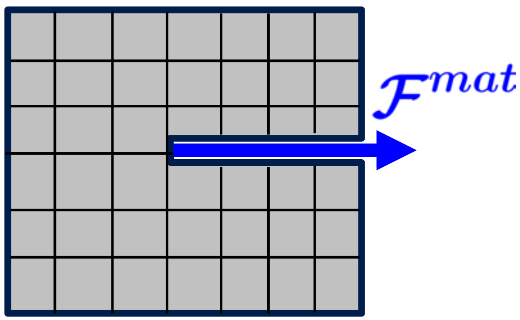
$$\nabla_X \cdot \boldsymbol{\Sigma}^T + \mathbf{B} = 0$$

$$\boldsymbol{\Sigma} = \psi \mathbf{I} - \mathbf{F}^T \mathbf{P}$$

$\boldsymbol{\Sigma}$ Eshelby stress

$\mathbf{B}$ material body forces

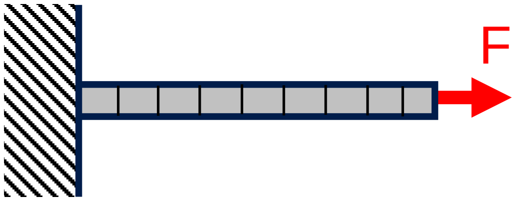
$\mathbf{F}$ deformation gradient



Computation of material forces

- integration of physical nodal forces

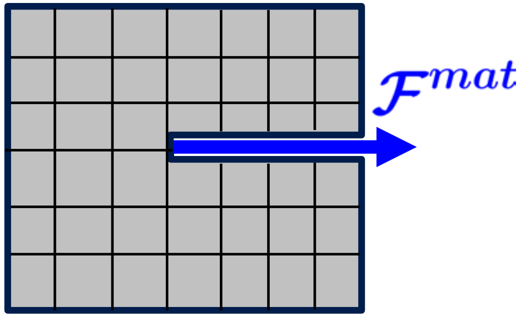
$$\mathbf{F} = \bigcup_{e=1}^{elem} \int_{\mathcal{B}} (\boldsymbol{\sigma} \cdot \nabla_x \mathbf{N} - \mathbf{b} \cdot \mathbf{N}) dv$$



$\boldsymbol{\sigma}$ Cauchy stress
$\mathbf{b}$ body forces
$\mathbf{N}$ shape functions

- integration of material nodal forces

$$\boldsymbol{\mathcal{F}}^{mat} = \bigcup_{e=1}^{elem} \int_{\mathcal{B}_0} (\boldsymbol{\Sigma} \cdot \nabla_X \mathbf{N} - \mathbf{B} \cdot \mathbf{N}) dV$$



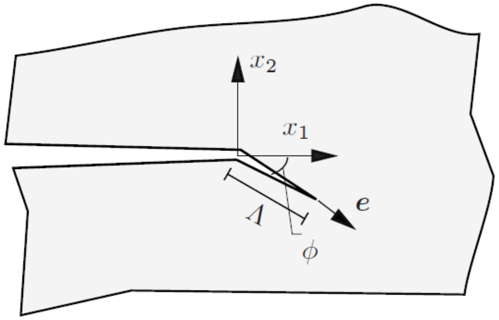
$\boldsymbol{\Sigma}$ Eshelby stress
$\mathbf{B}$ material body forces

Energy minimization technique

- crack direction

$$\Pi_{int}(\gamma+\Lambda,\tau) = \Pi_{int}(\gamma,\tau) - \mathcal{F}^{mat}\cdot\Lambda\mathbf{e} \;\rightarrow\; \min$$

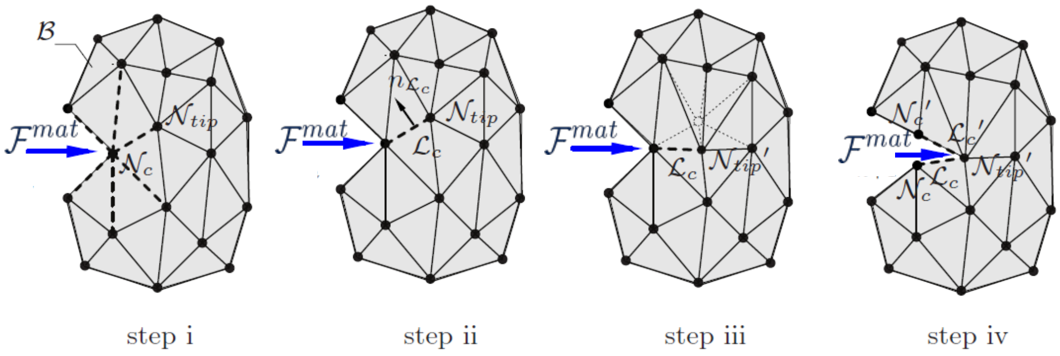
$$\phi = \operatorname{argmin}_{\phi\in[-\frac{\pi}{2},\frac{\pi}{2}]}(\Pi_{int}(\gamma,\tau) - \mathcal{F}^{mat}\cdot\Lambda\mathbf{e})$$



$\mathbf{e}$ unit crack direction vector
λ incremental crack size
ϕ crack kinking angle

- algorithm

$$\mathbf{e} = \frac{\mathcal{F}^{mat}}{|\mathcal{F}^{mat}|}$$



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uncertainty analysis $F_{UA}(M)$

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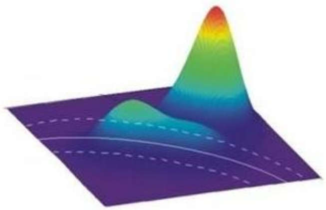
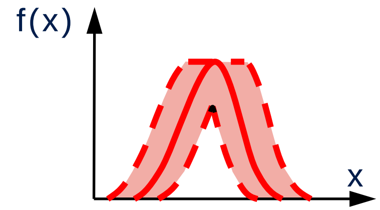
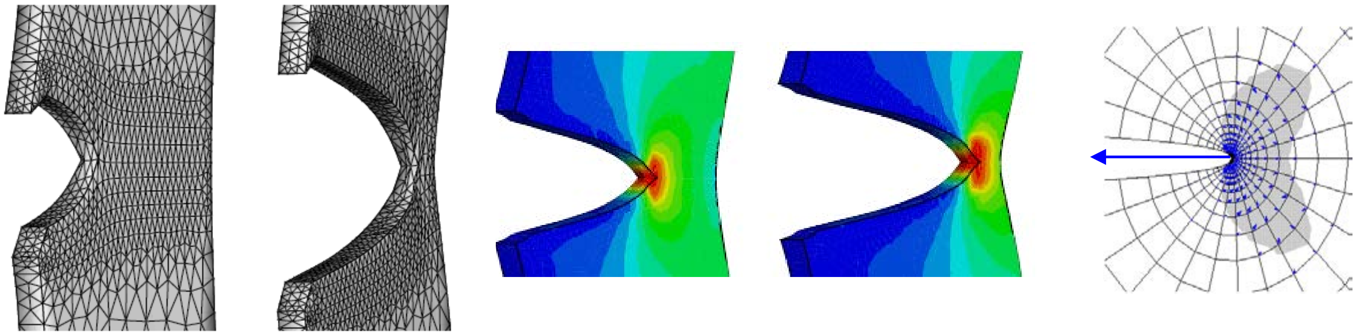


polymorphic uncertainty: fuzzy, stochastic, fuzzy-stochastic



finite element method
material force approach



Fail-safe design optimization

- multi-objective optimization task

$$\min \mathbf{f}(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \{f_1(\mathbf{x}_d, \tilde{\mathbf{p}}_a), f_2(\mathbf{x}_d, \tilde{\mathbf{p}}_a), ..., f_m(\mathbf{x}_d, \tilde{\mathbf{p}}_a)\}\,;$$

$$f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \mathcal{K}\left(\tilde{\gamma}_{cr}(\mathbf{x}_d, \tilde{\mathbf{p}}_a, \mathcal{F})\right);$$

subject to

$$g_k(\mathbf{x}_d, \tilde{\mathbf{p}}_a) \leq 0 \quad k = 1, 2, ..., p;$$

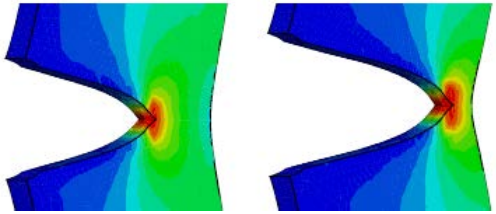
$$h_l(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = 0 \quad l = 1, 2, ..., q;$$

$$\mathcal{G}(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \frac{\partial \Pi_{int}}{\partial \gamma} = \mathcal{G}_c, \; \dot{\gamma} > 0;$$

$$\Pi_{int}(\gamma + \Lambda, t) = \Pi_{int}(\gamma, t) - \mathcal{F}^{mat} \cdot \Lambda \mathbf{e} \;\rightarrow\; \min!,$$

$$G^{\mathcal{M}}(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \nabla \cdot \boldsymbol{\Sigma}^t + \mathbf{B} = 0.$$

fracture mechanical analysis



$x_d \in \mathbb{R}^n$ design variables

$\tilde{p}_a \in \mathcal{F}(\mathbb{R}^{np})$ uncertain parameters

f_i objective function

g_k equality constraints

h_l inequality constraints

Objective function with uncertain input

- single objective function

$$f_i : \mathbb{R}^n \times \mathcal{F}(\mathbb{R})^{np} \rightarrow \mathcal{F}(\mathbb{R})$$

$$(\mathbf{x}_d, \tilde{\mathbf{p}}_a) \mapsto \tilde{z}_i, \quad i \in [1, m]$$

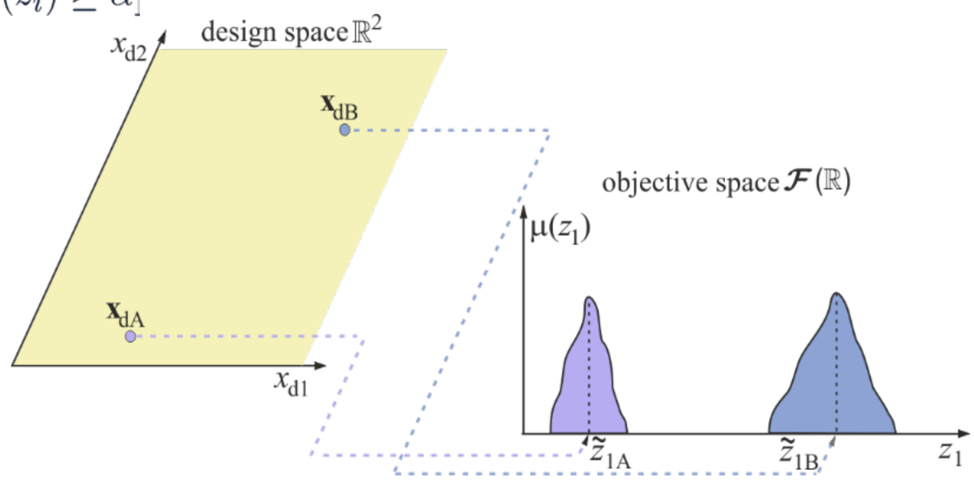
$\tilde{z}_i \in \mathcal{F}(\mathbb{R})$ fuzzy output quantity

- α-level discretization of $\tilde{z}_i$

$$\tilde{z}_i = (C_\alpha(\tilde{z}_i))_{\alpha \in (0,1]} = (\{[z_{il,\alpha}; z_{ir,\alpha}]\} \mid \alpha \in (0,1])$$

$$z_{il,\alpha} = \min\left[z_i \in C_\alpha(\tilde{z}_i),\ \mu_z(z_i) \geq \alpha\right]$$

$$z_{ir,\alpha} = \max\left[z_i \in C_\alpha(\tilde{z}_i),\ \mu_z(z_i) \geq \alpha\right]$$



Multiple objective functions with uncertain input

- m objective functions

$$\mathbf{f} : \mathbb{R}^n \times \mathcal{F}(\mathbb{R})^{np} \rightarrow \mathcal{F}(\mathbb{R}^m)$$

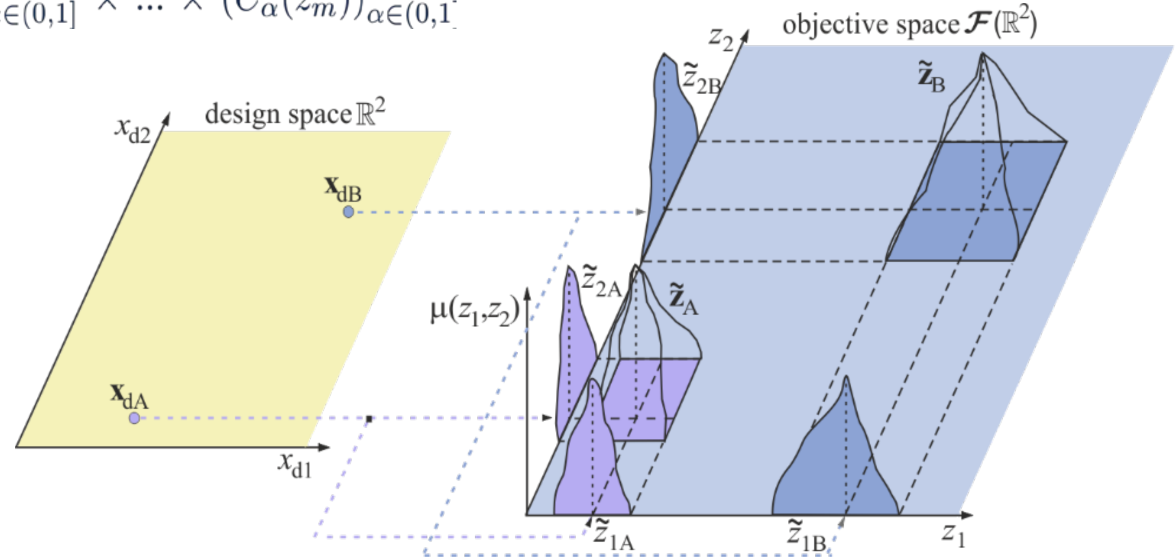
$$(\mathbf{x}_d, \tilde{\mathbf{p}}_a) \mapsto \tilde{\mathbf{z}}$$

$\tilde{\mathbf{z}} \in \mathcal{F}(\mathbb{R}^m)$ fuzzy objective vector

- α-level discretization of $\tilde{\mathbf{z}}$

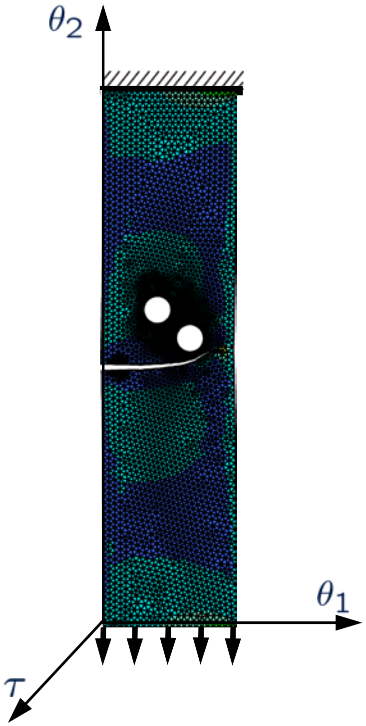
$$\tilde{\mathbf{z}} = \tilde{z}_1 \times \tilde{z}_2 \times \ldots \times \tilde{z}_m$$

$$\tilde{\mathbf{z}} = (C_\alpha(\tilde{z}_1))_{\alpha\in(0,1]} \, (C_\alpha(\tilde{z}_2))_{\alpha\in(0,1]} \times \ldots \times (C_\alpha(\tilde{z}_m))_{\alpha\in(0,1]}$$



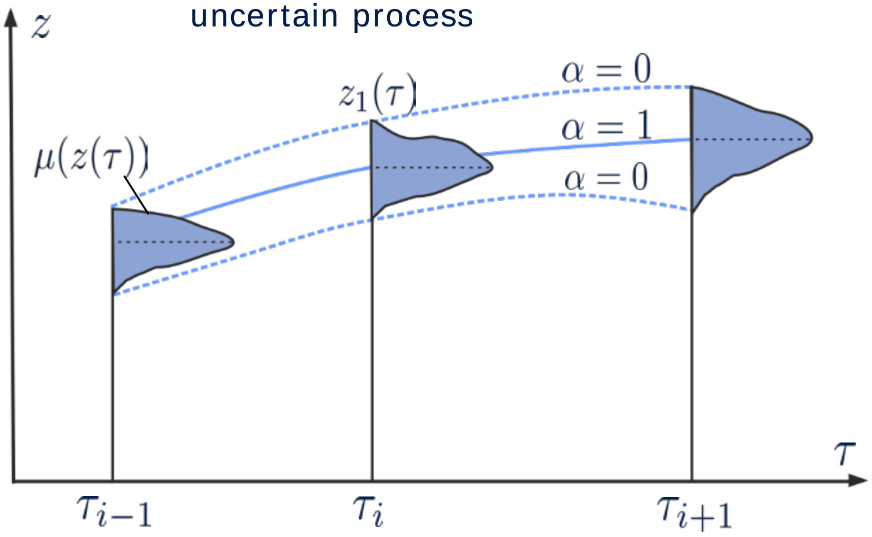
Uncertain output as a function

- uncertainty in time and space
  ⇩
  fracture



- uncertain functions
  - time dependent
  - dependent on spatial coordinates

uncertain process



Uncertain output – crack propagation

- uncertain crack propagation curve

$$\tilde{\gamma}_{cr} = (C_\alpha(\tilde{\gamma}_{cr}))_{\alpha \in (0,1]} \qquad C_\alpha(\tilde{\gamma}_{cr}) = \left\{\gamma_{cr} \in \mathbb{E}^3 : \ \mu_z \geq \alpha\right\}$$

- trajectory

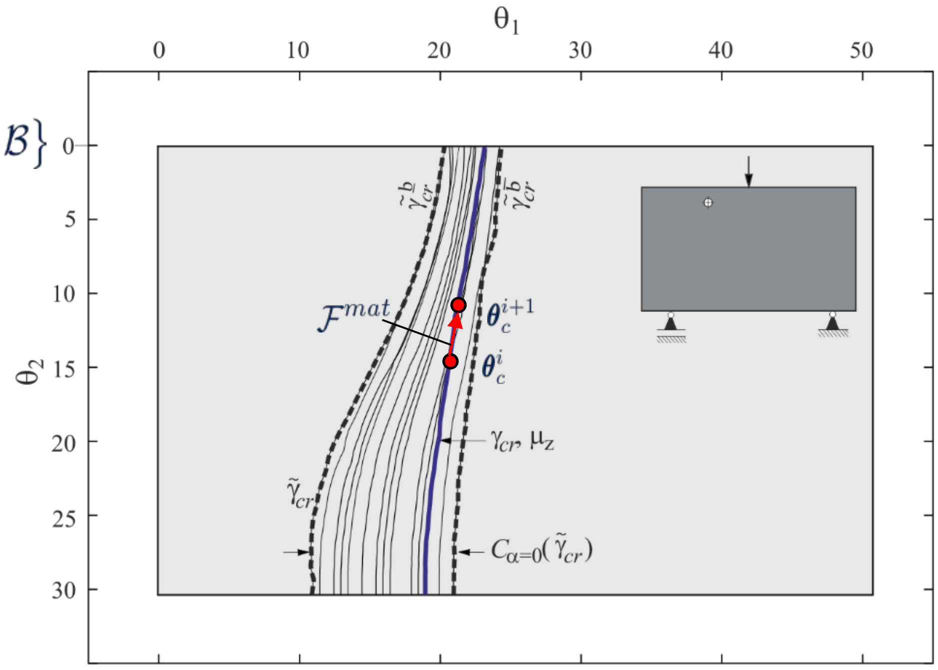
$$\gamma_{cr} = \left\{\boldsymbol{\theta}_c^1; ..., \boldsymbol{\theta}_c^i, ..., \boldsymbol{\theta}_c^{n_\theta} \mid \boldsymbol{\theta}_c^i = [\theta_1, \theta_2, \theta_3] \in \ \mathcal{B}\right\}$$

$$\gamma_{cr}^{\boldsymbol{\theta}_c} = \Lambda \mathrm{e}, \ \mathbf{e} = \frac{\mathcal{F}^{mat}}{|\mathcal{F}^{mat}|}$$

- bounding functions

$$\tilde{\gamma}_{cr}^{\underline{b}} = \min_{\boldsymbol{\theta}_c^{\hat{\bar{i}}j}|_j} \left[\gamma_{cr} \mid \gamma_{cr} \in C_{\alpha=0}(\tilde{\gamma}_{cr})\right]$$

$$\tilde{\gamma}_{cr}^{\overline{b}} = \max_{\boldsymbol{\theta}_c^{\hat{\bar{i}}j}|_j} \left[\gamma_{cr} \mid \gamma_{cr} \in C_{\alpha=0}(\tilde{\gamma}_{cr})\right]$$



$\mathcal{F}^{mat}$ material force $\quad \gamma_{cr}^{\boldsymbol{\theta}_c}$ crack direction vector

$\mathbf{e}$ Unit crack direction vector

Solution of optimization task

- application of information reducing measures

$$f(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \sum_{i=1}^{k} \sum_{j=1}^{l} w_{ij} \mathcal{M}_j(\tilde{z}_i) + w_u \mathcal{K}_u(\mathcal{M}_u(\tilde{\gamma}_{cr}))$$

$$\mathcal{M}_u(\tilde{\gamma}_{cr}) = \left\{ \tilde{\gamma}_{cr}^{\underline{b}}; \tilde{\gamma}_{cr}^{\bar{b}} \right\}$$

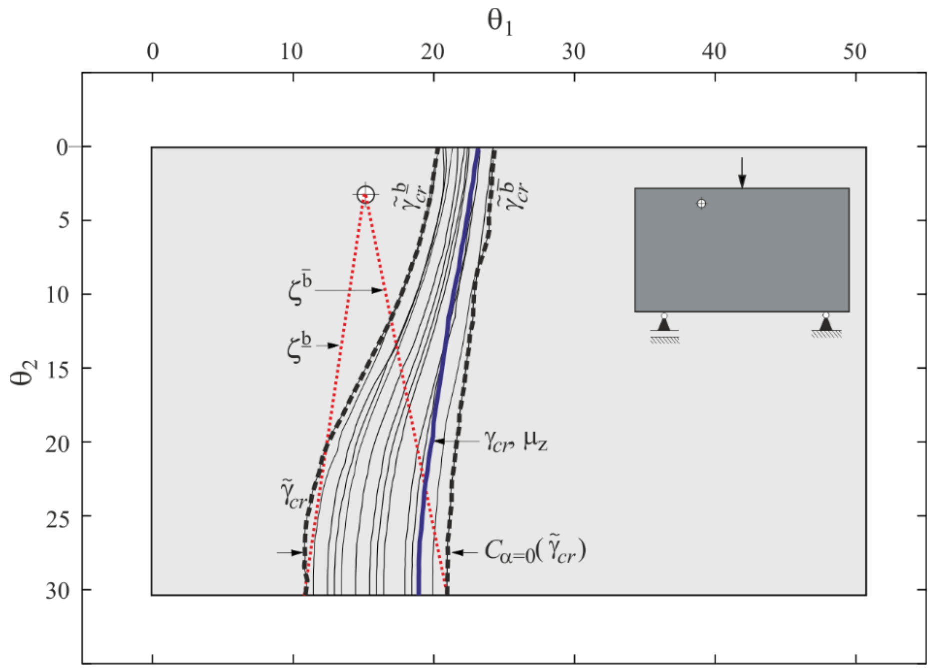
- assessment of crack propagation

$$\mathcal{K}_u(\mathcal{M}_u(\tilde{\gamma}_{cr})) = \sum_{l=1}^{n_\theta} P\left[d_{\mathbb{E}}\left(\tilde{\gamma}_{cr}^{\underline{b}}; \zeta^{\underline{b}}\right)\right] + \sum_{l=1}^{n_\theta} P\left[d_{\mathbb{E}}\left(\tilde{\gamma}_{cr}^{\bar{b}}; \zeta^{\bar{b}}\right)\right]$$

- boundary conditions for $\zeta^{\underline{b}}$, $\zeta^{\bar{b}}$

$$\zeta^{\bar{b}\,0} = \boldsymbol{\theta}_c^0; \quad \boldsymbol{\theta}_c^0 = \min\left[\boldsymbol{\theta}_c \mid \boldsymbol{\theta}_c \in S_{\alpha=0}(\tilde{p}_a)\right]$$

$$\zeta^{\bar{b}\,n_\theta} = \boldsymbol{\theta}_c^{n_\theta}; \quad \boldsymbol{\theta}_c^{n_\theta} = \chi_{Ai}$$



P penalty function $\quad d_{\mathbb{E}}$ Euclidean distance metric

$\zeta^{\underline{b}}, \zeta^{\bar{b}}$ aspired crack propagations

Solution of optimization task – multiple arrester

- application of information reducing measures

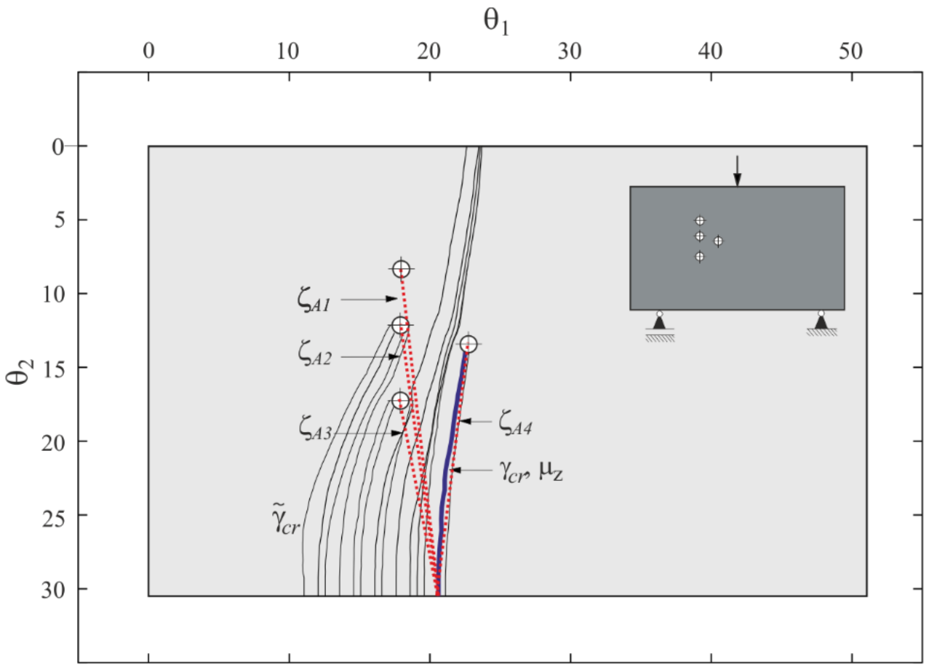
$$f(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \sum_{i=1}^{k} \sum_{j=1}^{l} w_{ij} \mathcal{M}_j(\tilde{z}_i) + w_u \mathcal{K}_u(\mathcal{M}_u(\tilde{\gamma}_{cr}))$$

- assessment of crack propagation

$$\mathcal{K}_u(\mathcal{M}_u(\tilde{\gamma}_{cr})) = \sum_{j=1}^{n_\gamma} \sum_{i=1}^{n_c} \left[\sum_{l=1}^{n_\theta} P\left[d_{\mathbb{E}}\left(\gamma_{cr\,j}; \zeta_{Ai}\right)\right] \right]$$

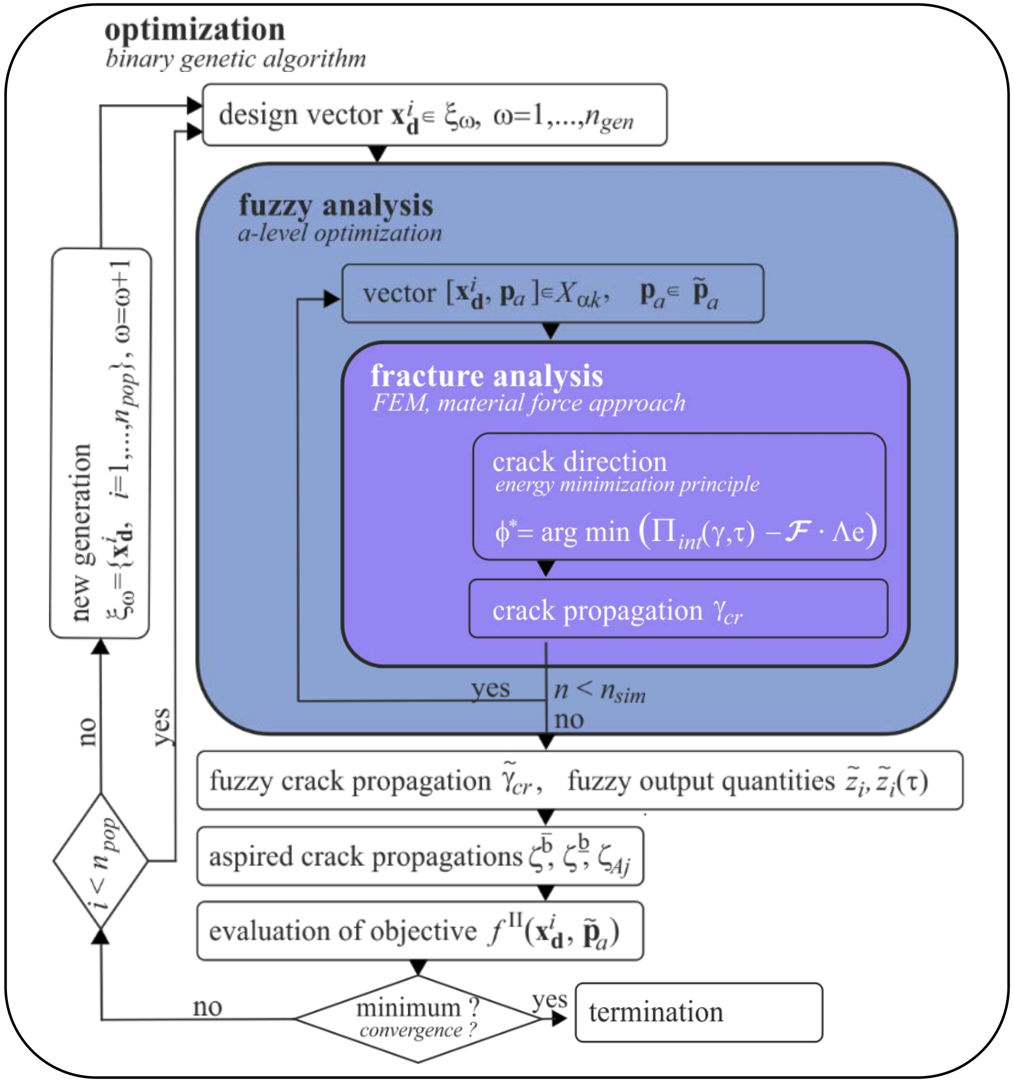
- boundary conditions for ζ_{Ai}

$$\zeta^{Ai,0} = \boldsymbol{\theta}_c^0; \quad \boldsymbol{\theta}_c^0 \in S_{\alpha=0}(\tilde{p}_a)$$
$$\zeta^{Ai,n_\theta} = \boldsymbol{\theta}_c^{n_\theta}; \quad \boldsymbol{\theta}_c^{n_\theta} = \chi_{Ai}$$



P penalty function $d_{\mathbb{E}}$ Euclidean distance metric
ζ^{Ai} aspired crack propagation

Numerical realization



Example 1 – small strains

- material: elastic, concrete

  ν = 0.18

  E = 38000 MPa

  $\mathcal{G}_c$ = 1.0 N/mm

- uncertain parameter

  $\tilde{\theta}_1^{cr}$ = <11.0,16.0, 21.0> cm

- design variables

  a = [3.175, 8.255] cm, Δa = 2.54 cm

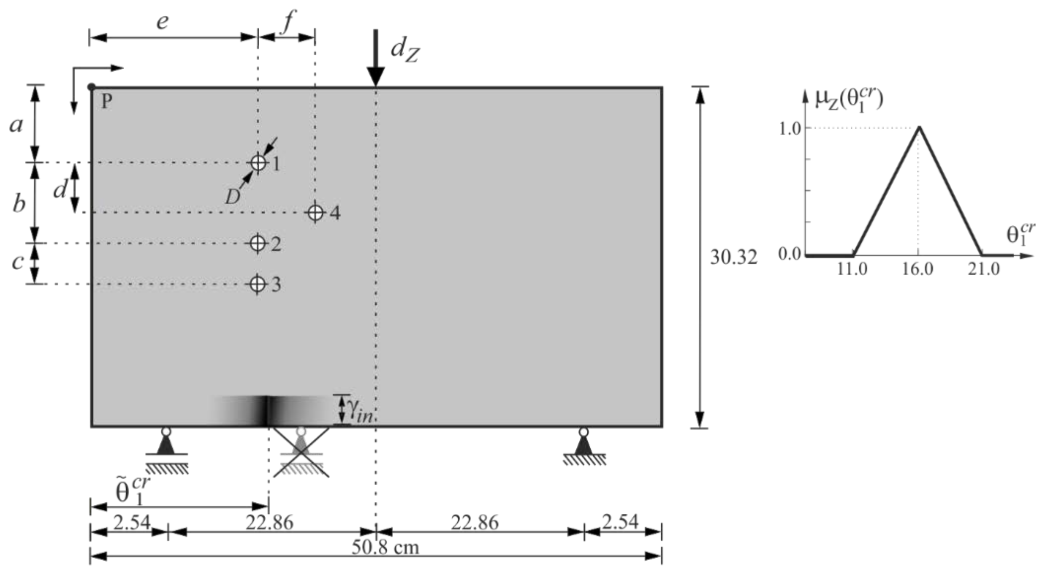
  b = [2.54, 5.08] cm, Δb = 1.27 cm

  c = [2.54, 5.08] cm, Δc = 1.27 cm

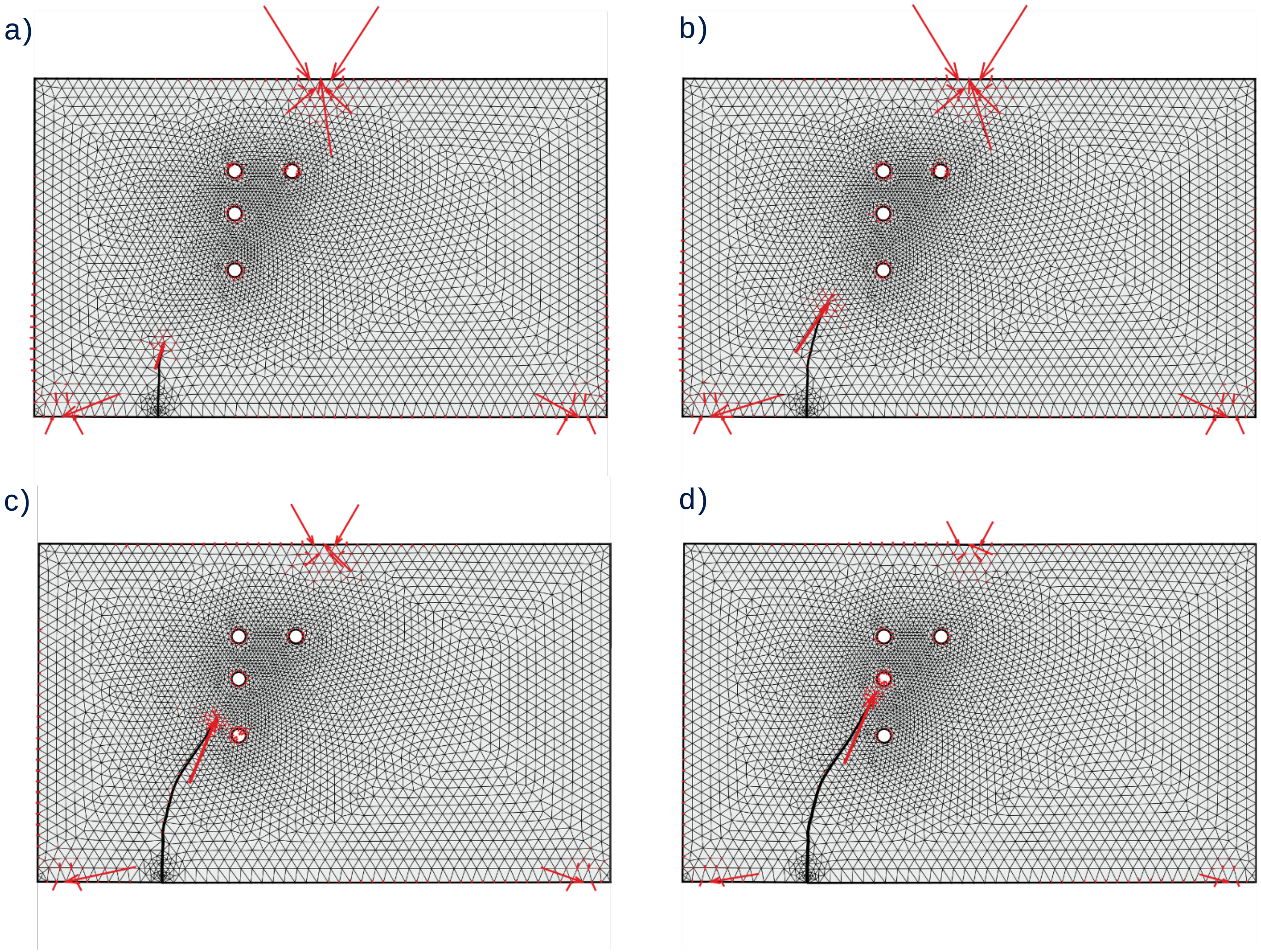
  d = [0, 10.16] cm, Δd = 2.54 cm

  e = [2.54, 27.94] cm, Δe = 5.08 cm

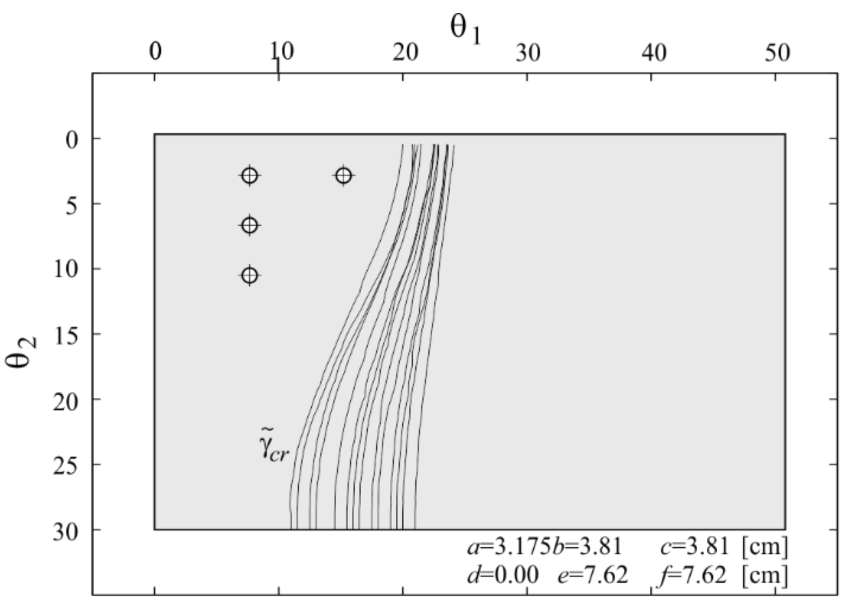
  f = [2.54, 10.16] cm, Δf = 2.54 cm γ_{in} = 1.254 cm



Deterministic crack propagation



Uncertain crack propagation

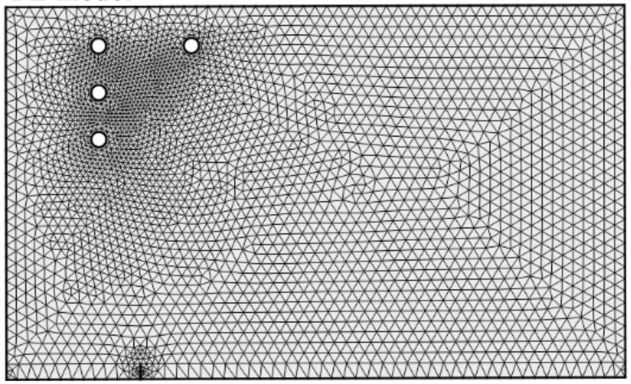


design configuration

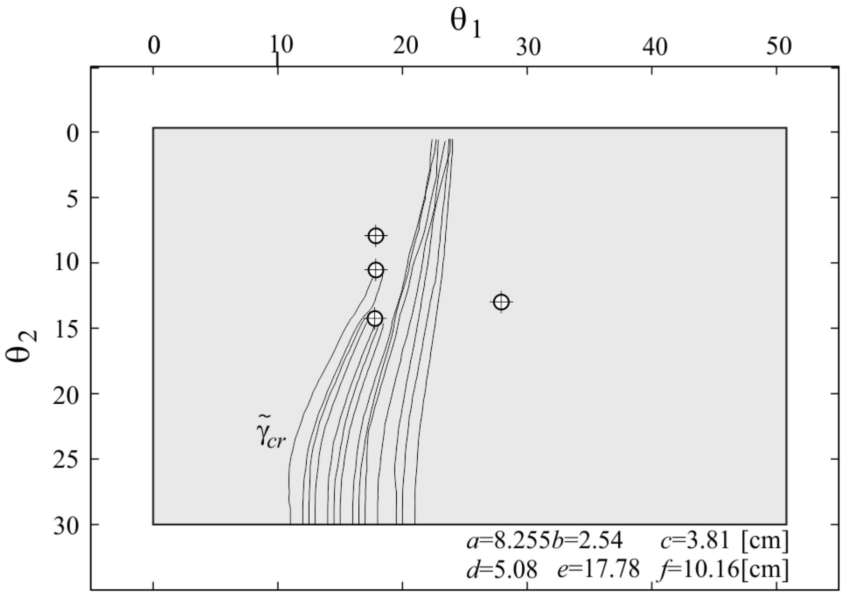
$\chi_{A1,1} = 7.62$ $\chi_{A2,1} = 7.62$ $\chi_{A3,1} = 7.62$ $\chi_{A4,1} = 15.24$ [cm]

$\chi_{A1,2} = 3.175$ $\chi_{A2,2} = 6.985$ $\chi_{A3,2} = 10.79$ $\chi_{A4,2} = 3.175$ [cm]

FE model



Uncertain crack propagation

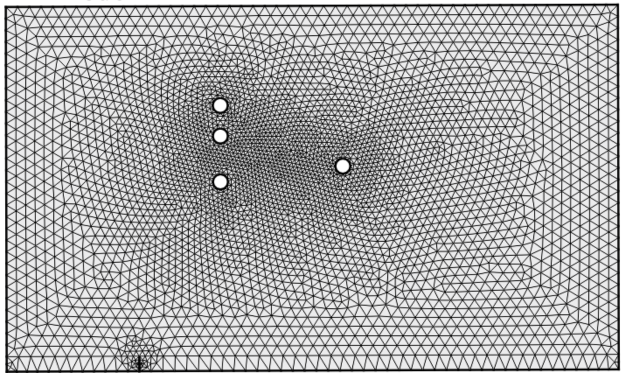


design configuration

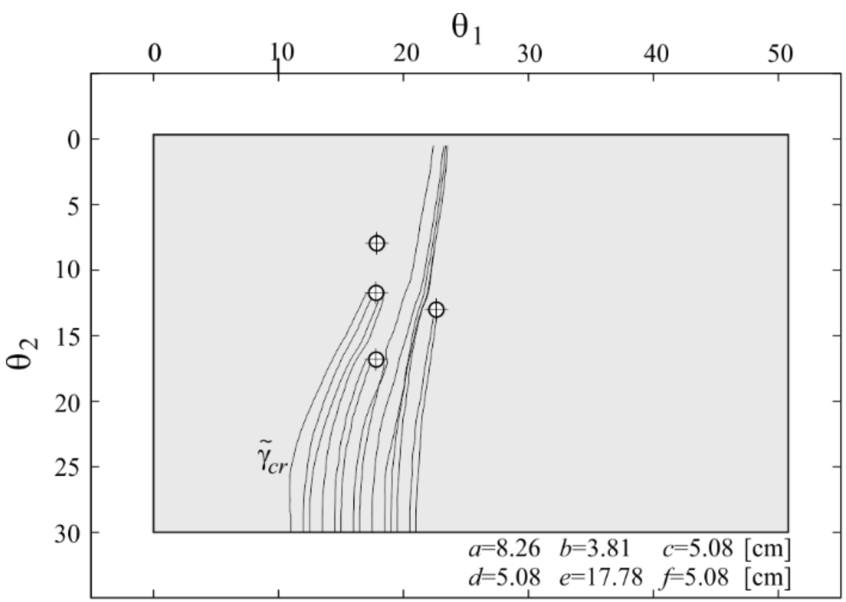
$\chi_{A1,1} = 17.78$ $\chi_{A2,1} = 17.78$ $\chi_{A3,1} = 17.78$ $\chi_{A4,1} = 27.94$ [cm]

$\chi_{A1,2} = 8.255$ $\chi_{A2,2} = 10.79$ $\chi_{A3,2} = 14.60$ $\chi_{A4,2} = 13.33$ [cm]

FE model



Uncertain crack propagation

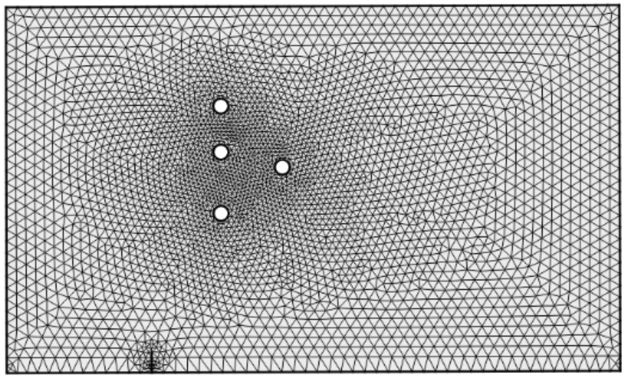


design configuration

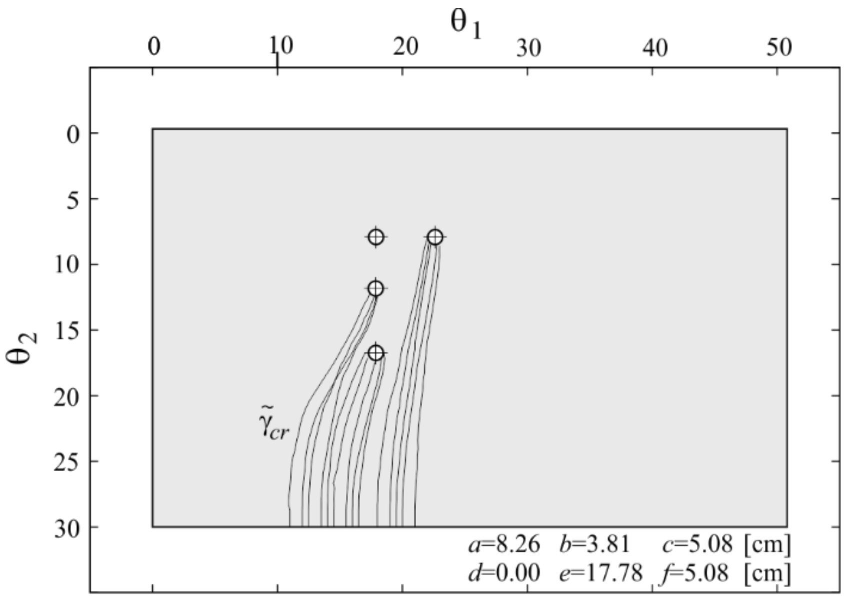
$\chi_{A1,1} = 17.78$ $\chi_{A2,1} = 17.78$ $\chi_{A3,1} = 17.78$ $\chi_{A4,1} = 22.86$ [cm]

$\chi_{A1,2} = 8.26$ $\chi_{A2,2} = 12.06$ $\chi_{A3,2} = 17.14$ $\chi_{A4,2} = 13.33$ [cm]

FE model



Uncertain crack propagation

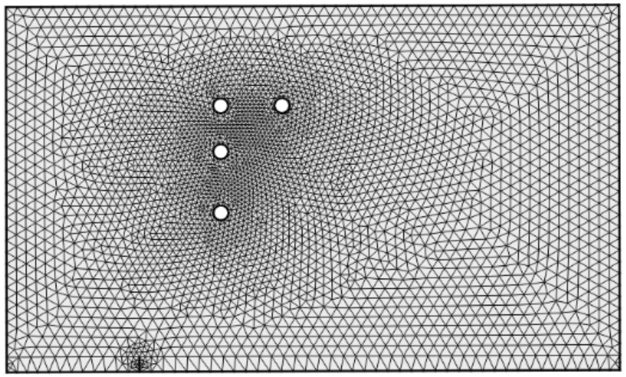


design configuration

$\chi_{A1,1}$= 17.78 $\chi_{A2,1}$= 17.78 $\chi_{A3,1}$= 17.78 $\chi_{A4,1}$= 22.86 [cm]

$\chi_{A1,2}$= 8.26 $\chi_{A2,2}$= 12.06 $\chi_{A3,2}$= 17.14 $\chi_{A4,2}$= 8.26 [cm]

FE model



Conclusions

- fail-safe optimal design method
- coupled approach of optimization, fuzzy analysis and fracture analysis
- uncertainties in crack initiation and propagation
- failure modeling in FE framework,
  discrete fracturing and configurational forces
- optimization algorithm learns the features of the uncertain crack propagation
- identification of optimal configuration of crack limiting elements
- prevention from undesired crack growth