TECHNISCHE UNIVERSITÄT DRESDEN

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Institute for Structural Analysis

# Numerical Simulation of Wooden Structures with Polymorphic Uncertainty in Material Properties

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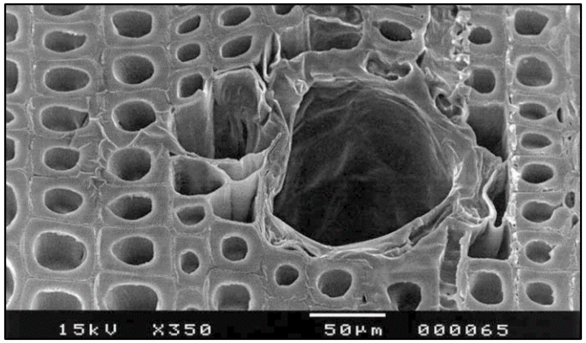
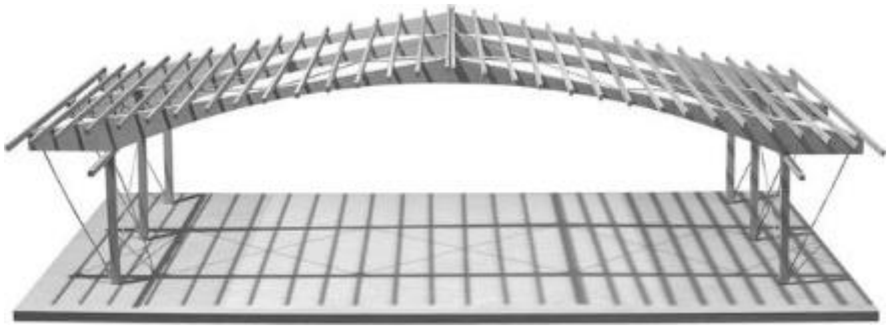
## Outline

## Motivation

- wood underlays large variation in material parameters
  - product of nature
  - discontinuities
  - dependencies to environmental conditions
- realistic representation of structural capabilities with respect to material uncertainties
- sustainable use of construction material



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## Motivation

Rafter and tie beam joint

- design according to DIN EN 1995
- cutting depth $t_v$

$$\frac{\sigma_{c,\alpha,d}}{f_{c,\alpha,d}} \leq 1 \text{ with } \sigma_{c,\alpha,d} = \frac{F_{c,\alpha,d}}{A_F} \text{ and } A_F = \frac{t_v}{\cos\alpha} \cdot b$$

$$f_{c,\alpha,d} = \frac{f_{c,0,d}}{\sqrt{\left(\frac{f_{c,0,d} \cdot \sin^2\alpha}{2 f_{c,90,d}}\right)^2 + \left(\frac{f_{c,0,d} \cdot \sin\alpha\cos\alpha}{2 f_{v,d}}\right)^2 + \cos^4\alpha}}$$

- front wood length

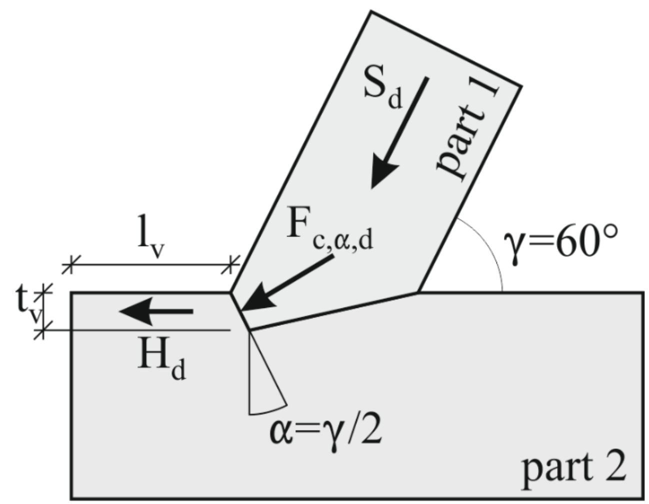
$$\frac{\tau_{v,d}}{f_{v,d}} \leq 1 \text{ with } \tau_{v,d} = \frac{H_d}{l_F \cdot b_{ef}} \text{ and } b_{ef} = k_{cr} \cdot b$$

- ultimate load

$$S_d = \min\left(\frac{f_{c,\alpha,d} \cdot A_F}{\cos\alpha}, \frac{f_{v,d} \cdot l_v \cdot b_{ef}}{\cos\gamma}\right)$$

$F_{c,\alpha,d} = S_d \cdot \cos(\alpha)$
$H_d = S_d \cdot \cos(\gamma)$

$h_1 = 160$
$h_2 = 200$
$b = 160$
$t_v = 30$
$l_v = 150$ [mm]

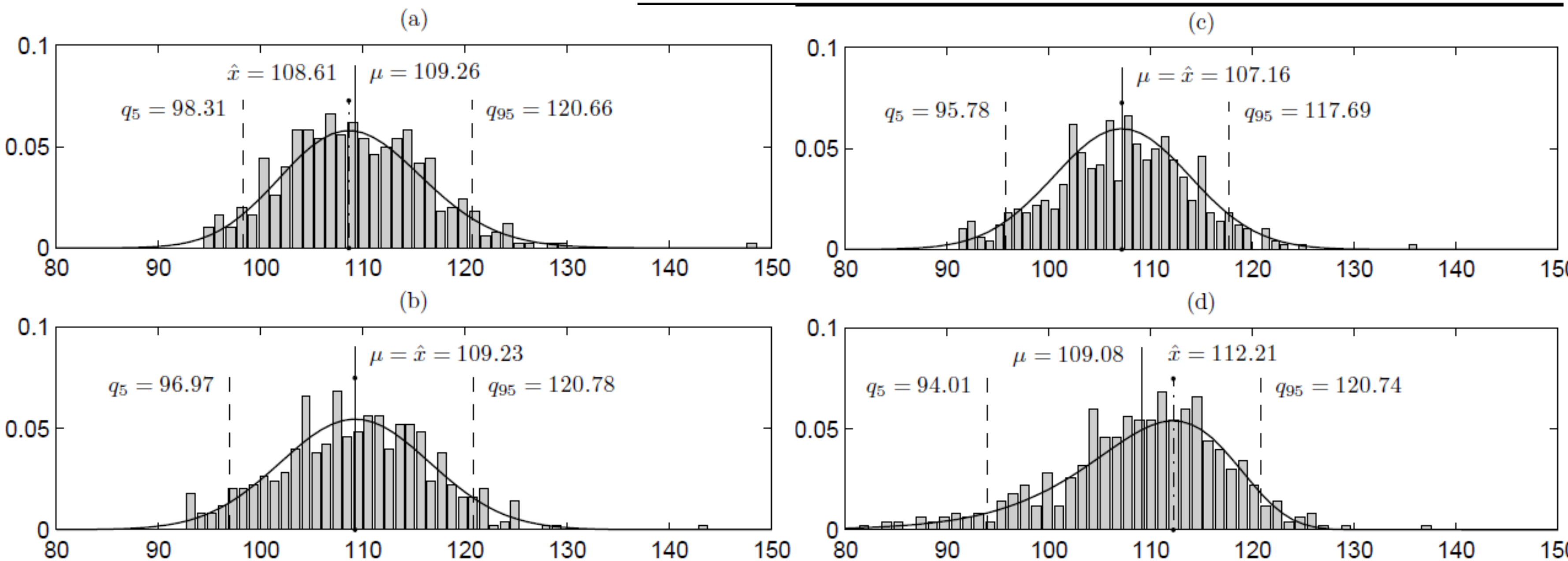


# Motivation

- influence of uncertain distribution parameter?

→ Fuzzy probability based random variable

| $f_v$-dist. | GUMBEL | Log-Normal | Normal | WEIBULL |
|---|---|---|---|---|
| Analysis | (a) | (b) | (c) | (d) |
| $S_d$-dist. | Log-Normal | Normal | Normal | WEIBULL |
| $\mathrm{Var}(\underline{S}_d)$ | 47.975 | 53.478 | 44.498 | 66.201 |
| parameter 1 | $\mu_u = 4.6918$ | $\mu = 109.23$ | $\mu = 107.16$ | $\theta = 112.64$ |
| parameter 2 | $\sigma_u = 0.0633$ | $\sigma = 7.3128$ | $\sigma = 6.6707$ | $k = 16.508$ |



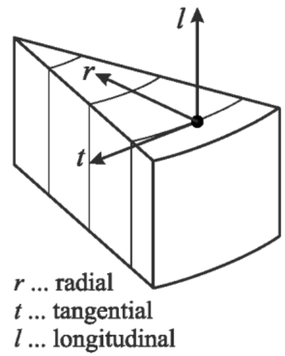
## Specimen tests

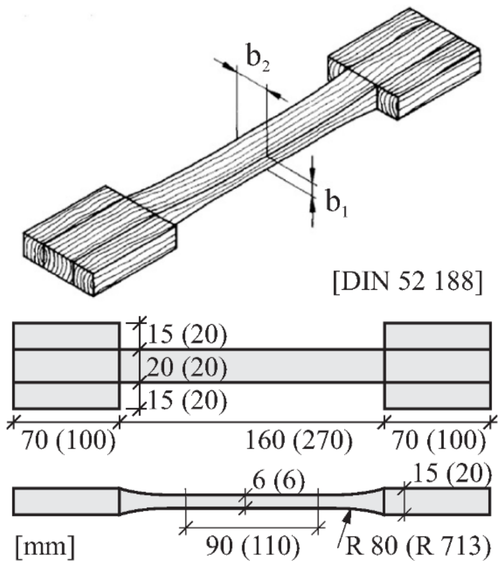
- Norway spruce specimen

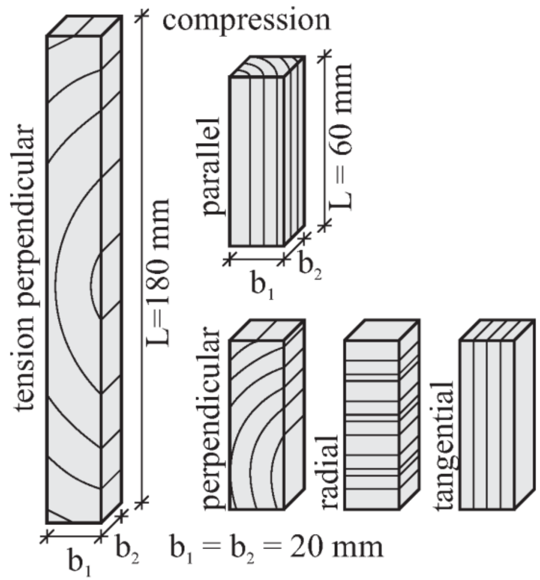
- material strengths
  $f_{t,90},\ f_{t,l},\ f_{c,r},\ f_{c,t},\ f_{c,l},\ f_v$

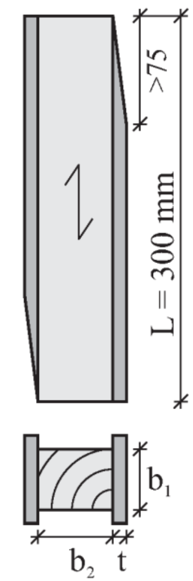
- elasticity moduli
  $E_r,\ E_t,\ E_l$

- all tests according to DIN and DIN EN
- 45/30 samples for each test mode
- standard climate conditions
  $T = 20 \pm 2°\mathrm{C},\ RH = 65 \pm \%$



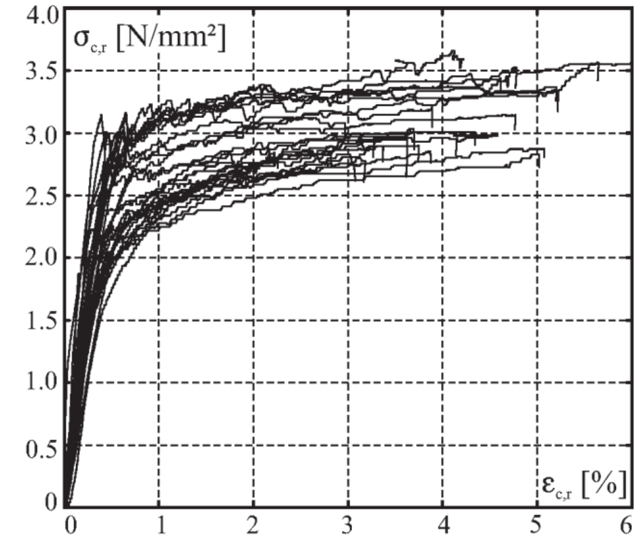



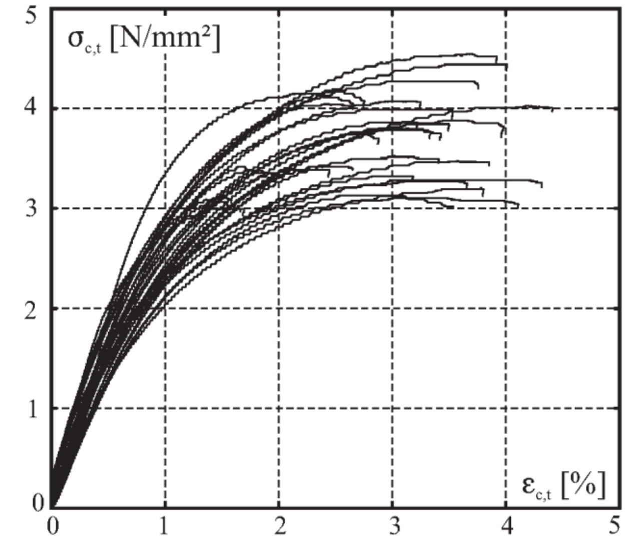



## Specimen tests

– considerably “good” circumstances
  broad variety of test results

– no strong correlation between
  geometric/climate conditions
  and test results



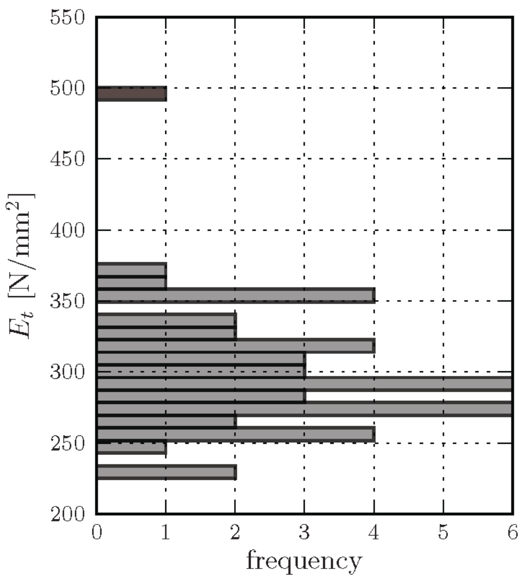



# Specimen tests

– preselection of outliers

$$l_{IQR} = 1.5(\bar{F}_X(\tfrac{3}{4}) - \bar{F}_X(\tfrac{1}{4}))$$
$$x_l = \bar{F}_X(\tfrac{1}{4}) - l_{IQR}$$
$$x_u = \bar{F}_X(\tfrac{3}{4}) + l_{IQR}$$

| parameter | $E_r$ | $E_t$ | $E_l$ | $f_{t,90}$ | $f_{t,l}$ | $f_{c,r}$ | $f_{c,t}$ | $f_{c,l}$ | $f_v$ |
|---|---|---|---|---|---|---|---|---|---|
| standard | DIN 52192 | DIN 52192 | DIN 52185 | EN 408 | DIN 52188 | DIN 52192 | DIN 52192 | DIN 52185 | EN 408 |
| samples | 41 | 44 | 28 | 30 | 30 | 45 | 45 | 30 | 30 |
| $\bar{m}$ $[N/mm^2]$ | 656 | 298 | 17132 | 2.64 | 121.64 | 3.09 | 3.64 | 43.60 | 5.77 |
| $\bar{\sigma}$ $[N/mm^2]$ | 107 | 36 | 2211 | 0.33 | 18.20 | 0.23 | 0.43 | 2.07 | 0.73 |



## Polymorphic uncertain structural analysis

– characterizing function $\xi(\cdot)$ for precise set $A \subseteq \mathbb{R}$

$$\xi_A : \mathbb{R} \to \{0;1\}, x \mapsto \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

– extension to non-precise data set

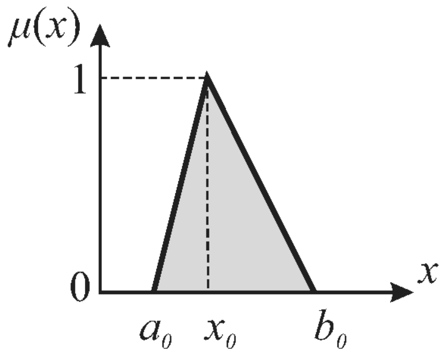
$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X, \mu_{\tilde{A}}(x) \geq 0\}$$

– characterizing function $\xi(\cdot)$ becomes membership function $\mu(\cdot)$

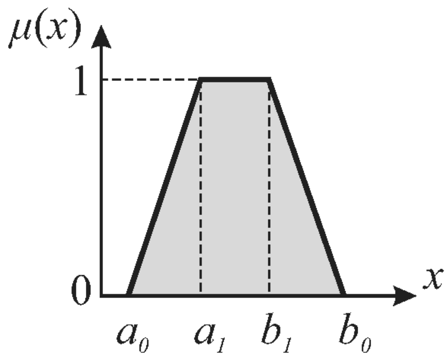
$$\mu : \mathbb{R} \to [0,1]$$

$$\exists\, x_0 \in \mathbb{R} : \mu(x_0) = 1$$

$$A_\alpha := \{x \in \mathbb{R} \mid \mu_{\tilde{A}}(x) \geq \alpha\} = [a_\alpha, b_\alpha]$$




$$\tilde{x} = \langle a_0, x_0, b_0 \rangle$$




$$\tilde{x} = \langle a_0, a_1, b_1, b_0 \rangle$$

# Polymorphic uncertain structural analysis

– random variable $X$
cannot be described precisely

– introduction of fuzzy probability

$$\hat{P} = (P_\alpha)_{\alpha \in (0;1]}$$

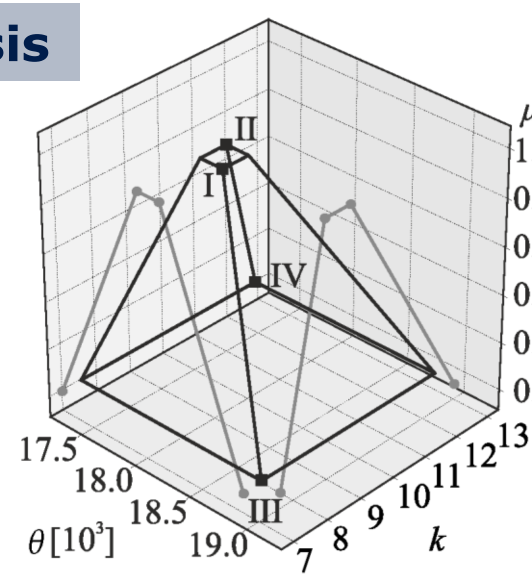
and fuzzy probability distribution

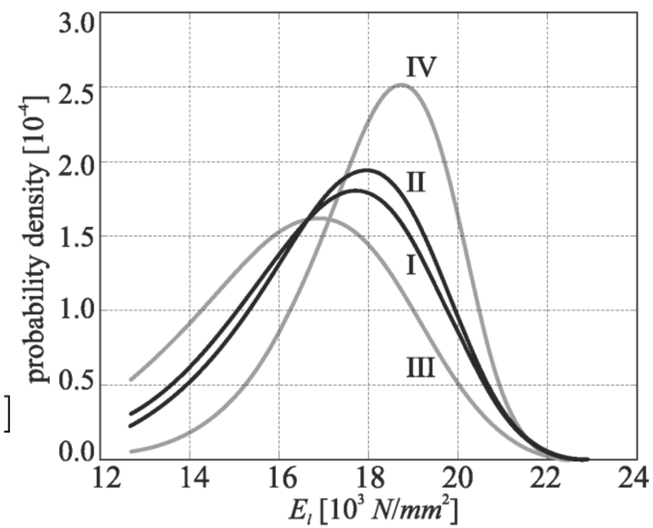
$$\hat{P}_X = ((P_X)_\alpha)_{\alpha \in (0;1]}$$

– representation as fuzzy cumulative
distribution with uncertain parameters $\tilde{\theta}_i$

$$\hat{F}_X = ((F_X)_\alpha)_{\alpha \in (0,1]}$$

$$\hat{F}_X = (\{F_{\theta_1 \times \theta_2} \mid \theta_1 \in \tilde{\theta}_{1,\alpha}, \theta_2 \in \tilde{\theta}_{2,\alpha}\})_{\alpha \in (0,1]}$$



# Polymorphic uncertain structural analysis

– three loop computational model

fuzzy analysis

$\tilde{\theta}_i = (\tilde{\theta}_{1,i} \times \tilde{\theta}_{2,i})$

stochastic analysis

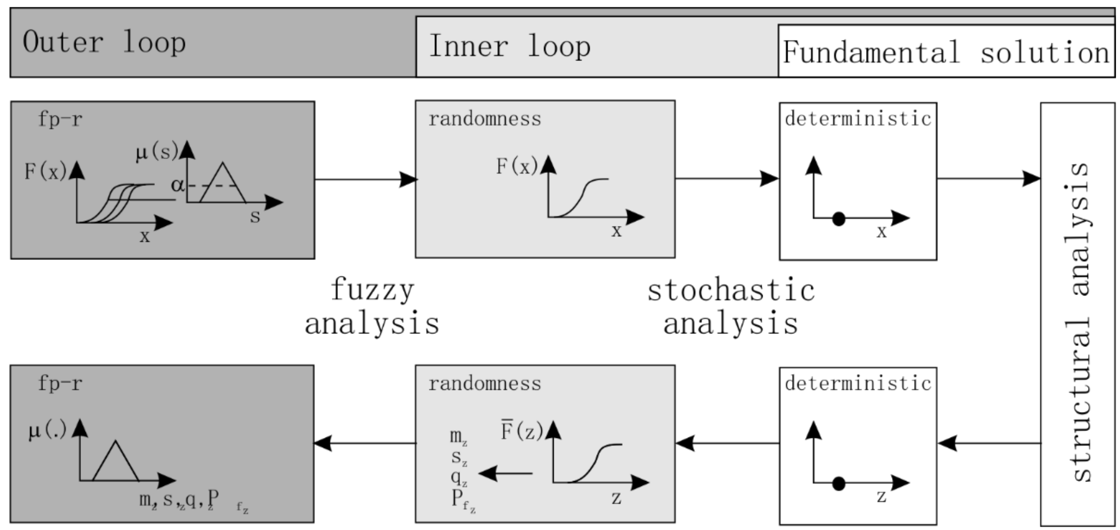
$F_{\theta_i}(x)$

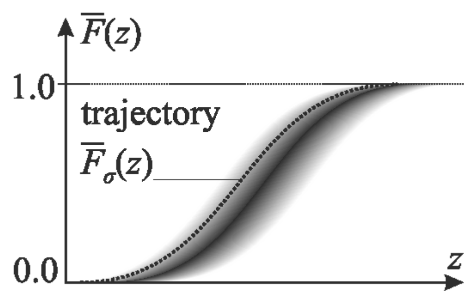
fundamental solution

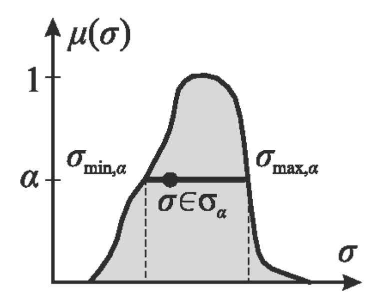
$f_Z : \mathbb{R}^n \rightarrow \mathbb{R}^m$

$\bar{F}_j(z)$

$\tilde{\sigma} = \langle \sigma_{\min,\alpha}, \sigma_{\max,\alpha} \rangle$



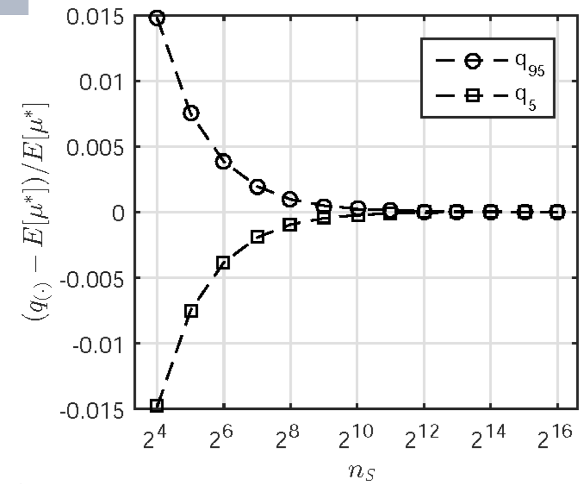
# Polymorphic uncertain structural analysis

– bounds for $A_0$

$$P_\theta(\vartheta_{\min} < \theta < \vartheta_{\max}) = (1 - \alpha_s)$$

$$\theta_{0,l} = \vartheta_{\min}$$

$$\theta_{0,r} = \vartheta_{\max}$$




– heuristic approach for bunch parameters, $A_1$

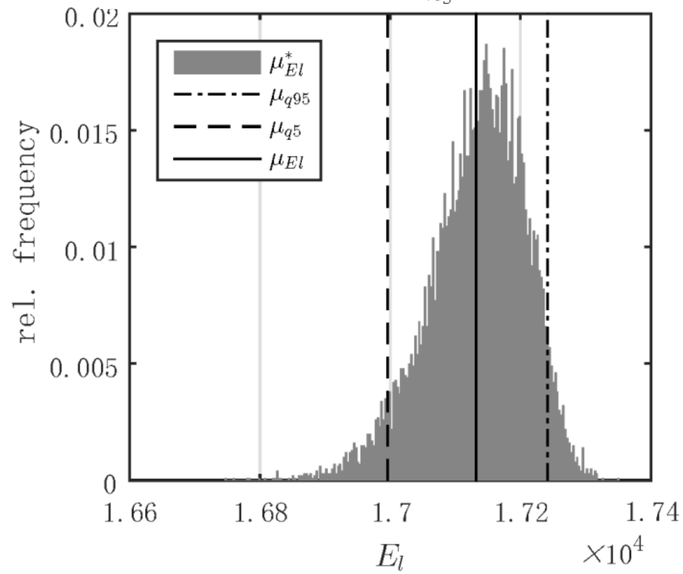
$$\bar{x} \in \bar{X}, P(\bar{X}) = F_X(\theta_1, \theta_2)$$

$$X_i^* = X \cup \{\bar{x}_i\} \; \forall \; \bar{x}_i \in \bar{X} \;,\; i = \{1, \ldots, n\}$$

$$\boldsymbol{\mu}^* = [\mathrm{E}[X_1^*], \ldots, \mathrm{E}[X_n^*]]$$

$$\boldsymbol{\sigma}^* = [\sqrt{\mathrm{Var}[X_i^*]} \ldots, \sqrt{\mathrm{Var}[X_n^*]}]$$

$$\mathrm{MoM} : \begin{cases} (\mu_{q5}, \sigma_{q5}) & \rightarrow (\vartheta_{1,\min_1}, \vartheta_{1,\min_2}) \\ (\mu_{q95}, \sigma_{q95}) & \rightarrow (\vartheta_{1,\max_1}, \vartheta_{1,\max_2}) \end{cases}$$



## Uncertain structural analysis

– timber board with 4 knotholes

$$t \times b \times l = 18 \times 150 \times 350\,mm$$

– computation of ultimate loading

$$p_u = \max(p)$$

– uncertain knothole size

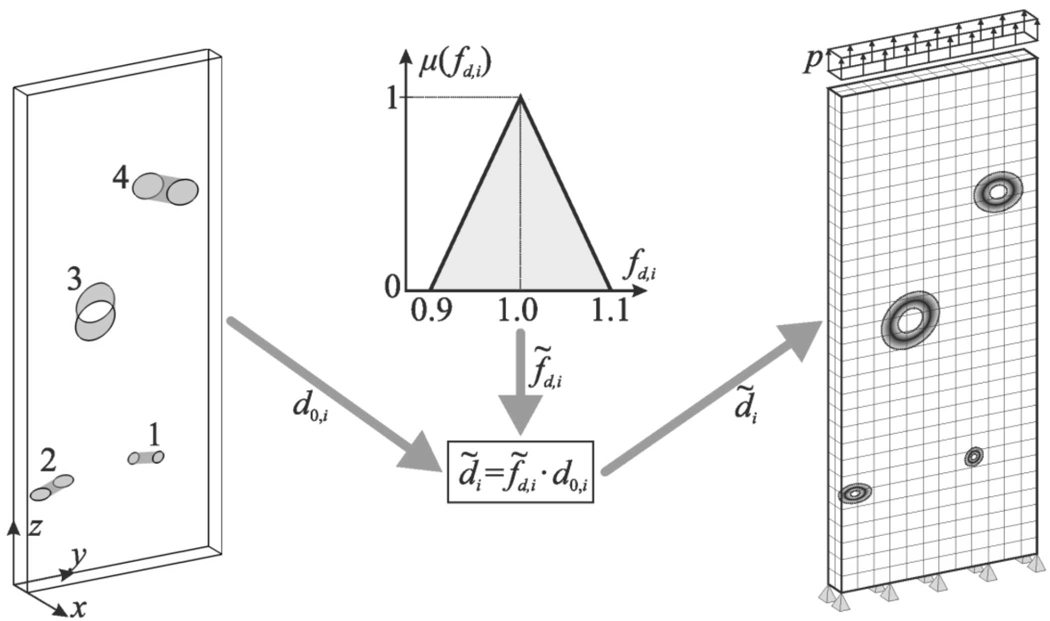
$$\widetilde{f}_{d,i} = \langle 0.9, 1.0, 1.1 \rangle$$

$$\widetilde{d}_i = \widetilde{f}_{d,i} \cdot d_{0,i}$$

$$i = \{1, 2, 3, 4\}$$

empty knotholes: $k_{type} = 1$

knotholes filled with branches: $k_{type} = 2$



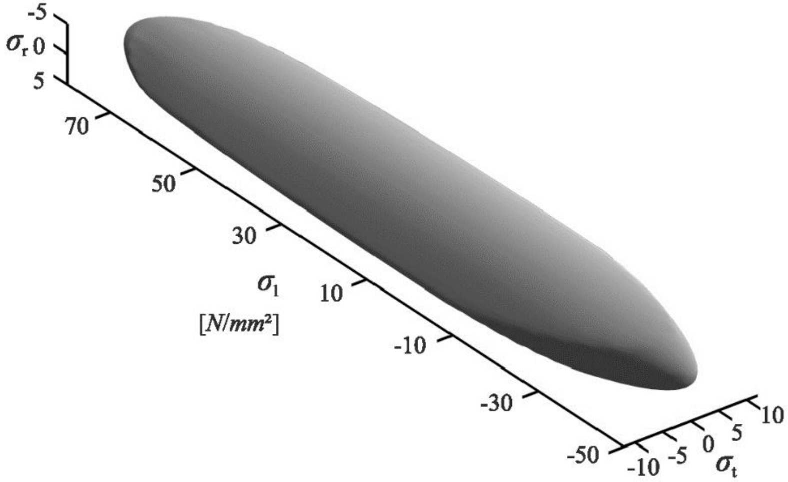
## Uncertain structural analysis

- multi-surface plasticity with hardening and softening rules in order to capture specific characteristics of wood
- seven failure modes
  - tension in radial direction
  - pressure in radial direction
  - tension in tangential direction
  - pressure in tangential direction
  - tension in longitudinal direction
  - pressure in longitudinal direction
  - shear failure
- TSAI-WU criterion for failure

  $$f(\underline{\sigma}) = \underline{a} : \underline{\sigma} + \underline{\sigma} : \underline{\underline{b}} : \underline{\sigma} \leq 1$$

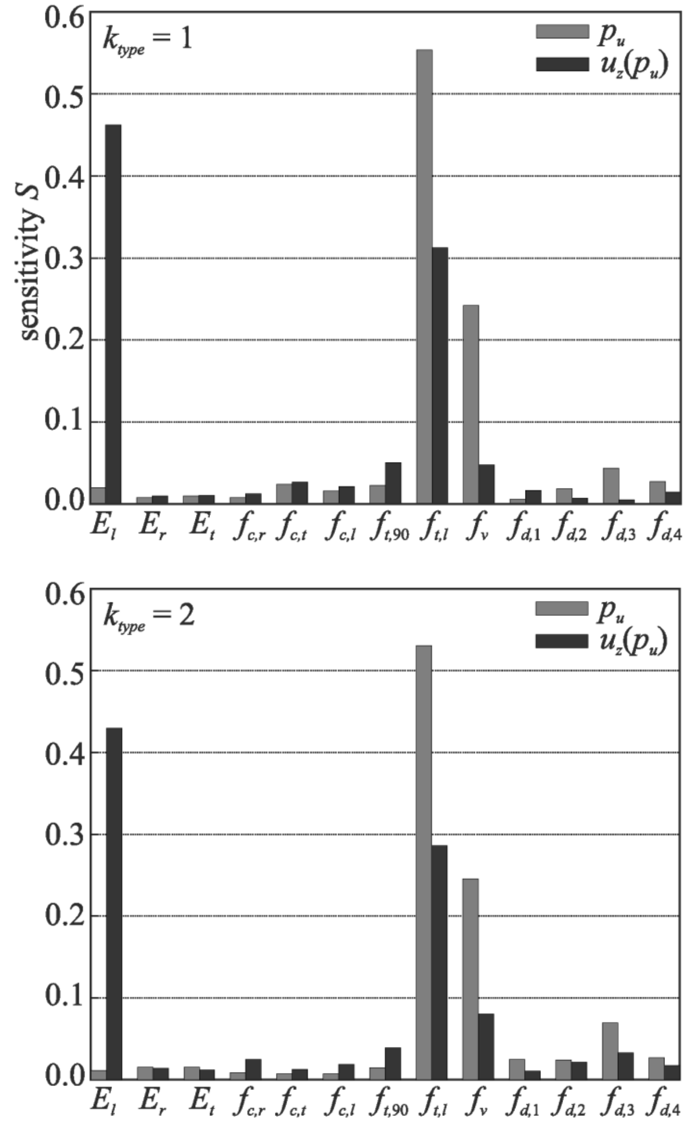
  with yield condition

  $$f(\underline{\sigma}) \leq 0$$



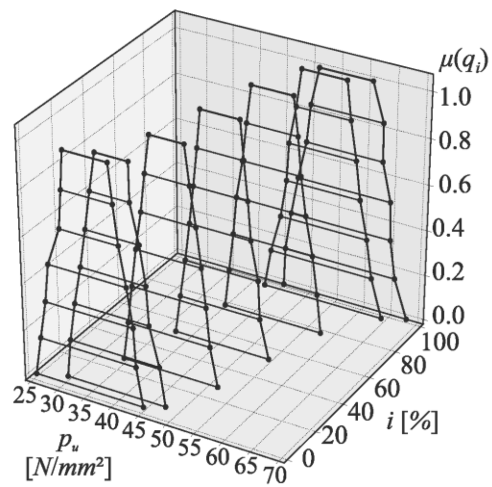
## Uncertain structural analysis

- evaluation on neural network
- preselection of input parameters based on sensitivity analysis
- evidently no significant influence of knothole size $\tilde{d}_i$
- trapezoidal interval distribution parameters $E_l,\ f_{t,l},\ f_v$
- triangular distribution parameters $E_r,\ E_t,\ f_{c,r},\ f_{c,t},\ f_{c,l},\ f_v$ $f_{d,1},\ f_{d,2},\ f_{d,3},\ f_{d,4}$

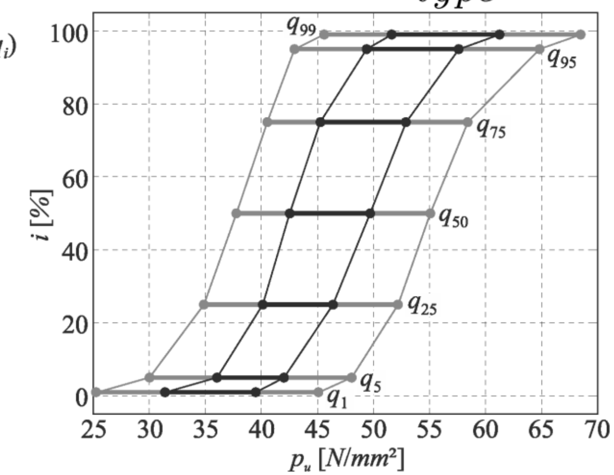


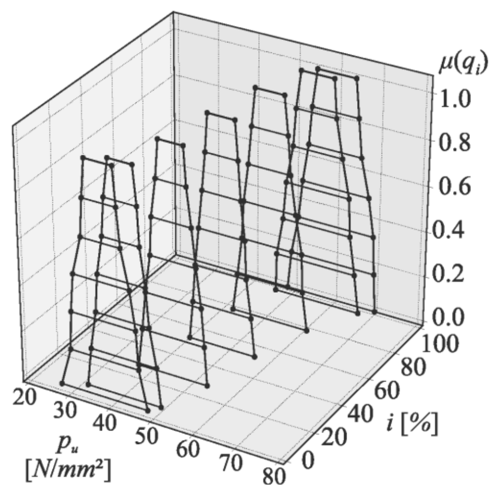
# Uncertain structural analysis

- 7 $\alpha$- levels
  $\alpha = \{0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1\}$
- computation of uncertain ultimate load $\tilde{p}_u$
- multiple quantile values $q_i$
  $i = \{1, 5, 25, 50, 75, 95, 99\}$
- apparently filling type minor effect to $\tilde{p}_u$

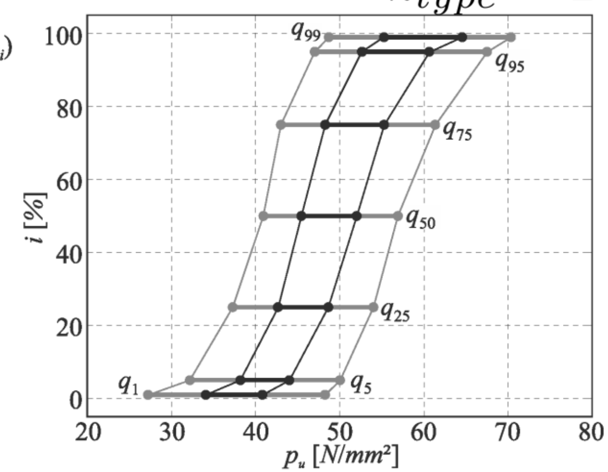



$k_{type} = 1$




$k_{type} = 2$



## Conclusion

- engineering related process to cope with material uncertainties
- use of generalized uncertainty model
- extension of information compared to deterministic design value

## Outlook

- further interpretation of uncertain result variables necessary in order to deduce design parameters / decisions
- no correlation of material properties considered
- geometrical distributed uncertain material parameters (fuzzy fields / stochastic fields)
- compare with p-boxes