Approximation Concepts for Fuzzy Analysis in Structural Dynamics

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Motivation

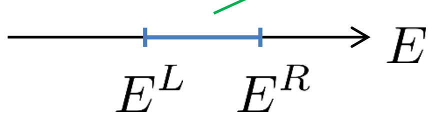
- Uncertainties are **unavoidable** in engineering
- Necessity of characterizing **structural performance** considering **uncertainty**

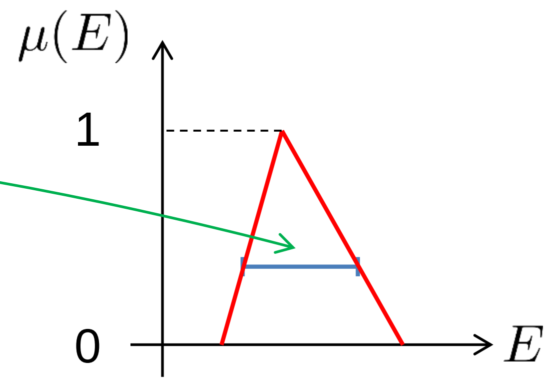
Traditional approach: **probability theory**

What happens in case uncertainty is due to **lack of knowledge**, **imprecision**, **vagueness**?

Epistemic uncertainty, described by **Non Traditional Uncertainty Models**

- Intervals and fuzzy sets



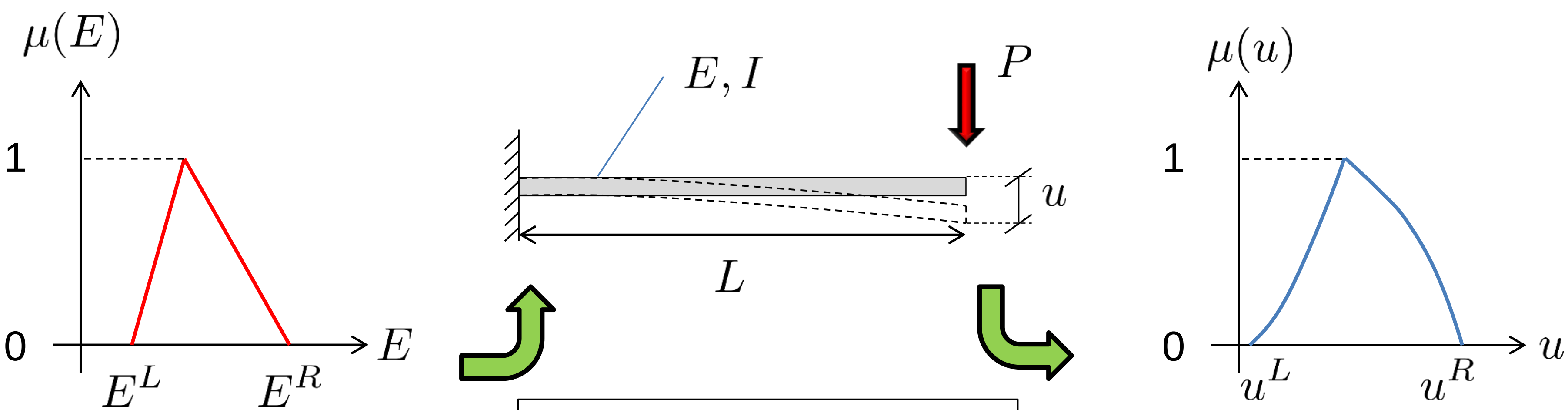


Interval: Extreme values known, no additional information

Fuzzy set: collection of intervals, μ represents, e.g. preference

Motivation

- <u>Fuzzy structural analysis</u>: **membership function of response**
- <u>Example</u>: Cantilever beam
  - Young's modulus E: fuzzy variable
  - Quantity of interest: tip displacement u

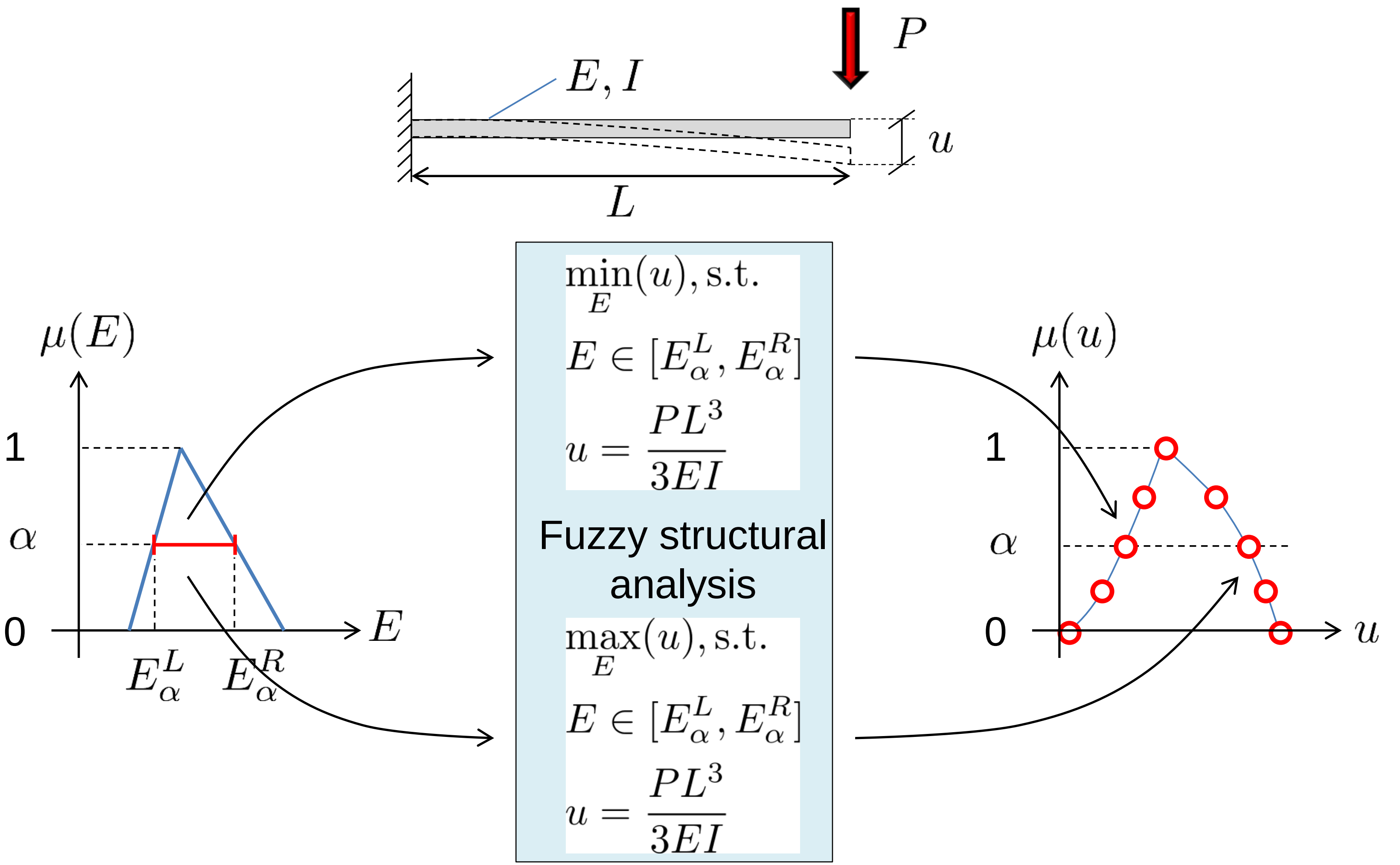


Fuzzy structural analysis

Valuable information about sensitivity

Motivation

- <u>Determination of membership function</u>: α ***–level discretization***



Motivation

- Determination of membership function: α ***–level discretization***

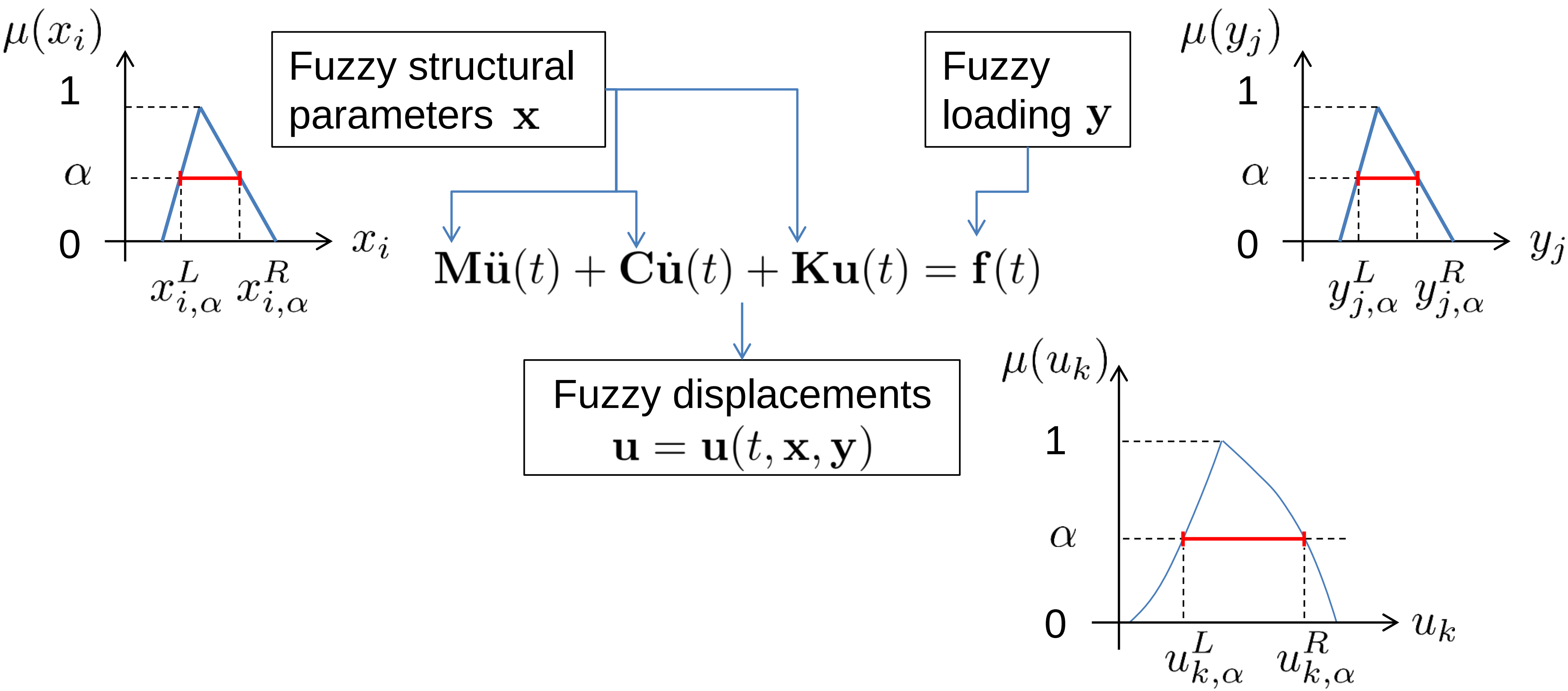
Can provide exact bounds, no dependency problem

Repeated structural analyses (optimization)

- This presentation
  - Strategies for performing approximate fuzzy structural analysis in linear dynamics
  - Focus on numerical efficiency (reduced number of full structural analyses)

Fuzzy Analysis: Linear Dynamics

- <u>Equilibrium equation</u>
  - Formulated within context of FE method
  - Classical damping matrix considered



Fuzzy Analysis: Linear Dynamics

- Equilibrium equation
  - $\mathbf{f}(t)$: periodic loading
  - Response of interest: steady-state displacement

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \sum_{l=1}^{N_f} \mathbf{f}_l(\mathbf{y}) e^{i\omega_l(\mathbf{y})t}$$

Steady-state displacement

$$\mathbf{u}_P = \sum_{l=1}^{N_f} \mathbf{H}(\omega_l(\mathbf{y}), \mathbf{x}) \mathbf{f}_l(\mathbf{y}) e^{i\omega_l(\mathbf{y})t}$$

Transfer function

$$\mathbf{H} = \sum_{r=1}^{N_m} \frac{\boldsymbol{\phi}_r(\mathbf{x})\boldsymbol{\phi}_r^T(\mathbf{x})}{\omega_r(\mathbf{x})^2 - \omega^2 + i2\xi_r\omega\omega_r(\mathbf{x})}$$

Fuzzy Analysis: Linear Dynamics

- Equilibrium equation
  - Response of interest: steady-state displacement

$$\mathbf{u}_P = \sum_{l=1}^{N_f} \mathbf{H}(\omega_l(\mathbf{y}), \mathbf{x})\mathbf{f}_l(\mathbf{y})e^{i\omega_l(\mathbf{y})t}$$

  - Challenge: calculation of transfer function and **spectral properties**

$$\left(\mathbf{K}(\mathbf{x}) - \omega_r^2(\mathbf{x})\mathbf{M}(\mathbf{x})\right)\boldsymbol{\phi}_r(\mathbf{x}) = \mathbf{0},\ \ r = 1, \ldots, N_m$$

Retained modes

  - Proposed strategy: approximation of transfer function

$$\mathbf{H}(\omega, \mathbf{x}) \approx \tilde{\mathbf{H}}(\omega, \mathbf{x})$$

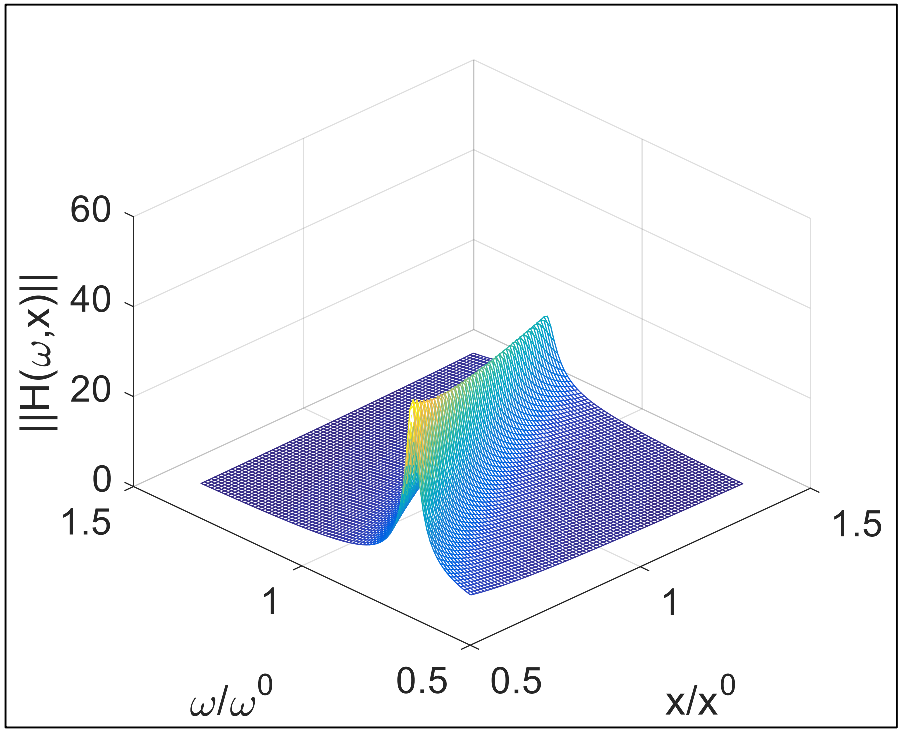
Approximation of intermediate quantities

Intervening variables

Fuzzy Analysis: Linear Dynamics

- <u>Approximation of transfer function</u>
  - Note linear approximation would not be applicable

$$\mathbf{H}(\omega, \mathbf{x}) \approx \tilde{\mathbf{H}}(\omega, \mathbf{x}) = \mathbf{H}^0 + \mathbf{H}_{,\omega}(\omega - \omega^0) + \sum_{i=1}^{N_p} \mathbf{H}_{,i}(x_i - x_i^0)$$



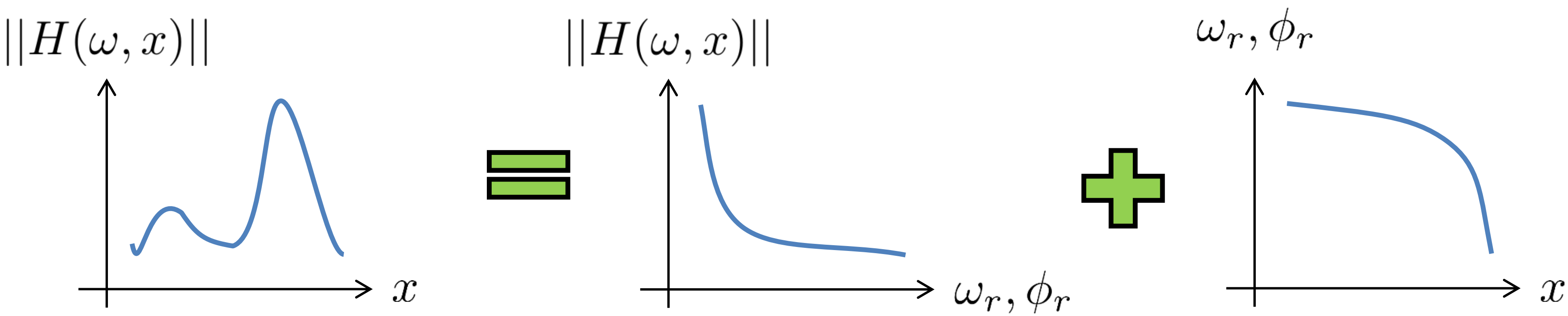
FRF for a single-degree-of-freedom system

Fuzzy Analysis: Linear Dynamics

- Approximation of transfer function
  - First concept: intermediate quantities*
    - Transfer function approximated in terms of spectral properties

$$\mathbf{H}(\omega,\mathbf{x}) \approx \tilde{\mathbf{H}}(\omega,\mathbf{x}) = \sum_{r=1}^{N_m} \frac{\tilde{\boldsymbol{\phi}}_r(\mathbf{x})\tilde{\boldsymbol{\phi}}_r^T(\mathbf{x})}{\tilde{\omega}_r(\mathbf{x})^2 - \omega^2 + i2\xi_r\omega\tilde{\omega}_r(\mathbf{x})}$$

$\tilde{\omega}_r(\mathbf{x}), \tilde{\boldsymbol{\phi}}_r(\mathbf{x})$: approximate spectral properties



*Haftka, R. & Gürdal, Z. Elements of Structural Optimization. Kluwer, 1992

Fuzzy Analysis: Linear Dynamics

- Approximation of transfer function
  - Second concept: intervening variables
    - Natural frequencies approximated considering **intervening variables** of exponential type

$$\omega_r \approx \tilde{\omega}_r = \omega_r^0 + \sum_{i=1}^{N_p} \frac{\partial \omega_r}{\partial I_{r,i}} \left(I_{r,i}\left(x_i\right) - I_{r,i}\left(x_i^0\right)\right)$$

$$\boxed{I_{r,i}(x_i) = x_i^{m_{r,i}}}$$

    - Exponent associated with intervening variable is selected based on **sensitivity analysis**

$$\left.\frac{\partial^2 \omega_r}{\partial x_i^2}\right|_{\mathbf{x}=\mathbf{x}^0} = \left.\frac{\partial^2 \tilde{\omega}_r}{\partial x_i^2}\right|_{\mathbf{x}=\mathbf{x}^0}, \; i = 1, \ldots, N_p$$

Fuzzy Analysis: Linear Dynamics

- Approximation of transfer function
  - Approximation for mode shapes: linear interpolation*

$$\boldsymbol{\phi}_r \approx \tilde{\boldsymbol{\phi}}_r = \boldsymbol{\phi}_r^0 + \sum_{i=1}^{N_p} \frac{\partial \boldsymbol{\phi}_r}{\partial x_i} \left(x_i - x_i^0 \right)$$

  - Sensitivity of spectral properties
    - Required sensitivities calculated using approach** that requires only retained modes plus N_m matrix factorizations

$$\frac{\partial \omega_r}{\partial x_i},\ \frac{\partial^2 \omega_r}{\partial x_i^2},\ \frac{\partial \boldsymbol{\phi}_r}{\partial x_i},\ i = 1, \ldots, N_p,\ r = 1, \ldots, N_m$$

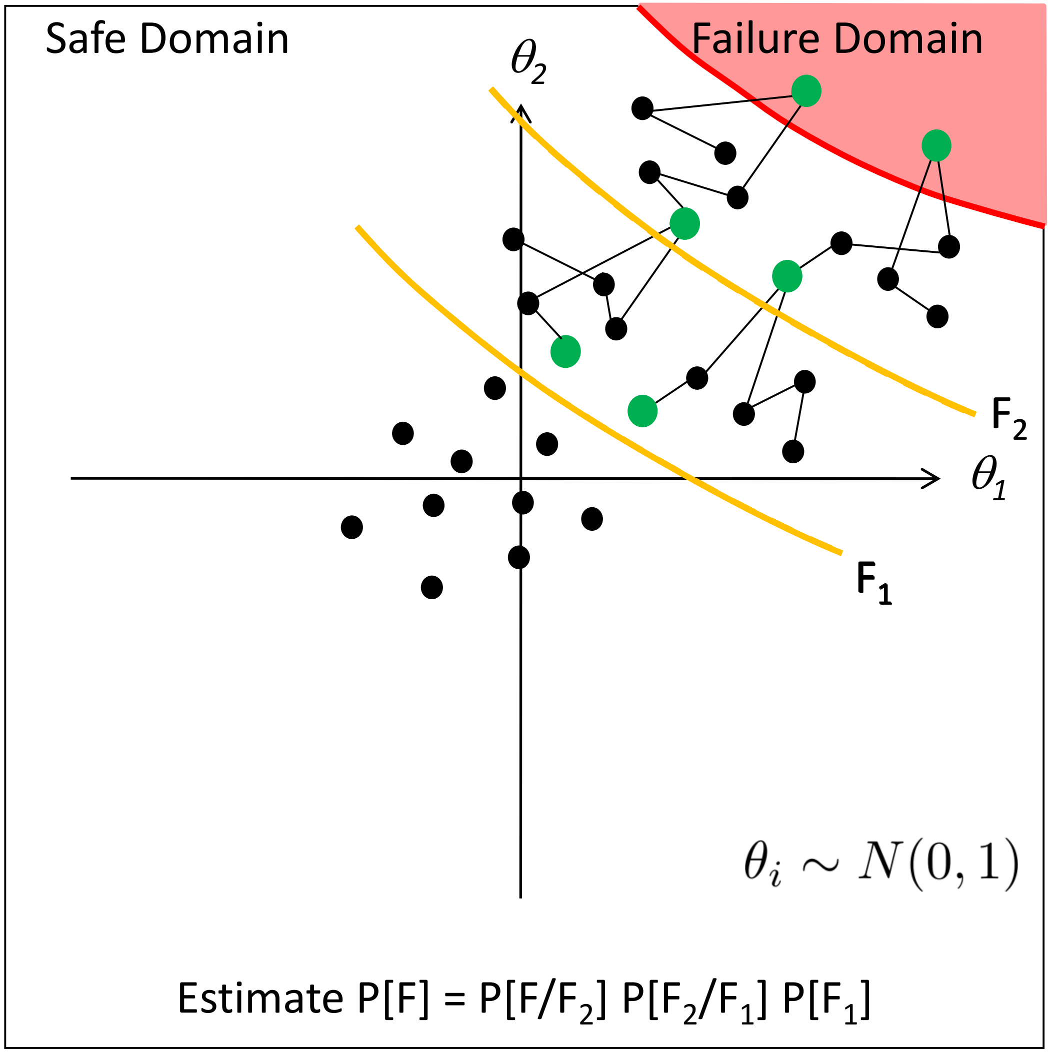
*Sim, J.; Qiu, Z. & Wang, X. Modal analysis of structures with uncertain-but-bounded parameters via interval analysis. Journal of Sound and Vibration , 2007, 303, 29 – 45
**Nelson, R. Simplified calculation of eigenvector derivatives AIAA Journal, 1976, 14, 1201-1205

Fuzzy Analysis: Linear Dynamics

- α-Level Optimization
  - Applied considering approximate transfer function

$$\mathbf{u}_P = \sum_{l=1}^{N_f} \tilde{\mathbf{H}}(\omega_l(\mathbf{y}), \mathbf{x})\mathbf{f}_l(\mathbf{y})e^{i\omega_l(\mathbf{y})t}$$

  - Subset simulation for optimization*
    - Simulation-based
    - Gradient-free



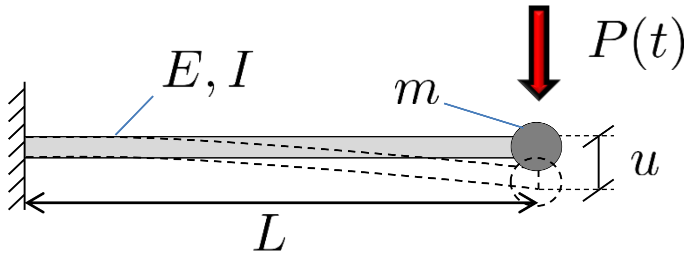
*Li, H. & Au, S. Design optimization using Subset Simulation algorithm Structural Safety, 2010, 32, 384-392

Fuzzy Analysis: Linear Dynamics

- <u>Summary of proposed approach</u>

1. Perform spectral analysis and sensitivity analysis (nominal value)

$$(\mathbf{x}^0, \mathbf{y}^0) \Rightarrow$$



$$\Rightarrow \omega_r^0,\ \boldsymbol{\phi}_r^0 \quad \frac{\partial \omega_r^0}{\partial x_i},\ \frac{\partial^2 \omega_r^0}{\partial x_i^2},\ \frac{\partial \boldsymbol{\phi}_r^0}{\partial x_i}$$

2. Determine exponent of intervening variable

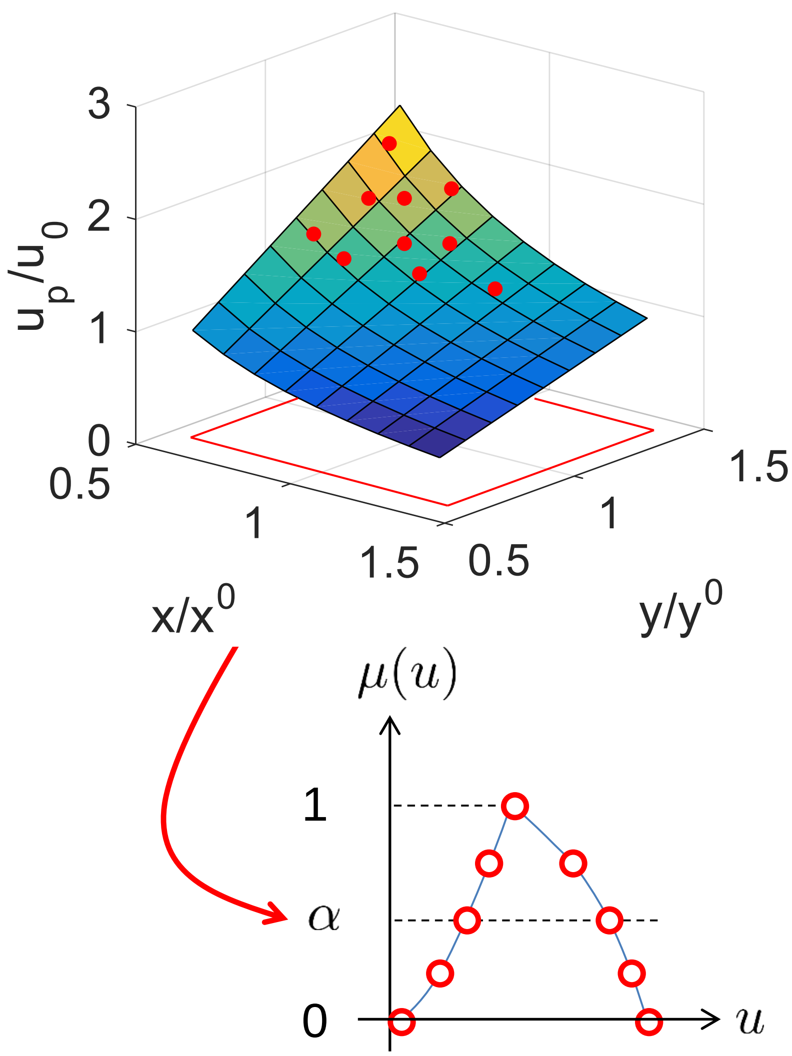
$$\left.\frac{\partial^2 \omega_r}{\partial x_i^2}\right|_{\mathbf{x}=\mathbf{x}^0} = \left.\frac{\partial^2 \tilde{\omega}_r}{\partial x_i^2}\right|_{\mathbf{x}=\mathbf{x}^0} \Rightarrow m_{r,i},\ I_{r,i}(x_i) = x_i^{m_{r,i}}$$

Fuzzy Analysis: Linear Dynamics

- <u>Summary of proposed approach</u>
  3. Construct proposed approximation

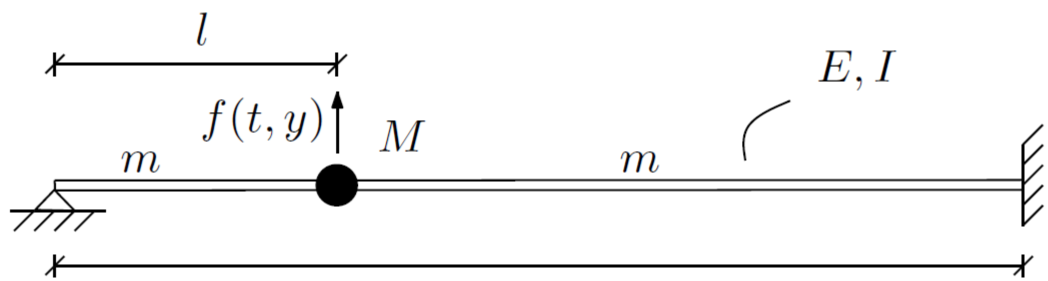
$$\mathbf{u}_P = \sum_{l=1}^{N_f} \tilde{\mathbf{H}}(\omega_l(\mathbf{y}), \mathbf{x})\mathbf{f}_l(\mathbf{y})e^{i\omega_l(\mathbf{y})t}$$

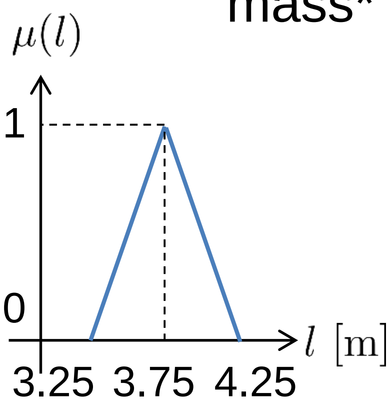
  4. For each α-level, determine extrema of response (Subset simulation for optimization)

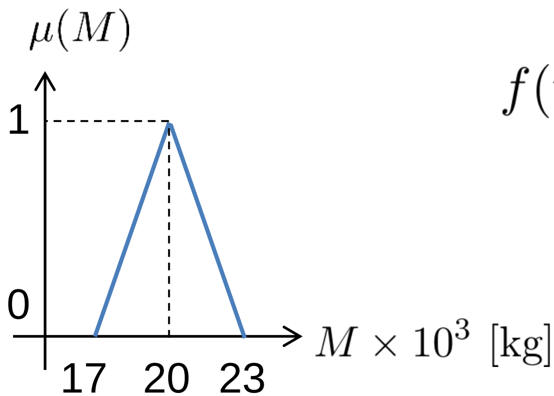


Examples

- <u>Example 1</u>
  - Steel beam subject to harmonic loading*
  - Distributed and point mass*

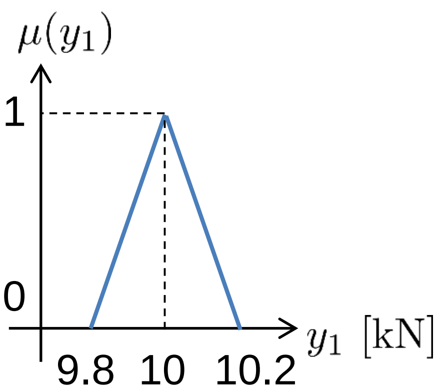


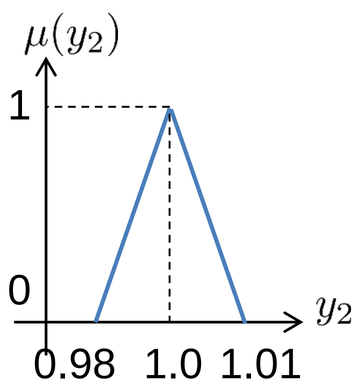




$$f(t,\mathbf{y}) = y_1 \cos(y_2 \omega_1 t) + y_1 \cos(y_2 \omega_2 t)$$

  - <u>Response of interest</u>: displacement of point mass
  - <u>Model</u>: 12 elements, 23 DOF

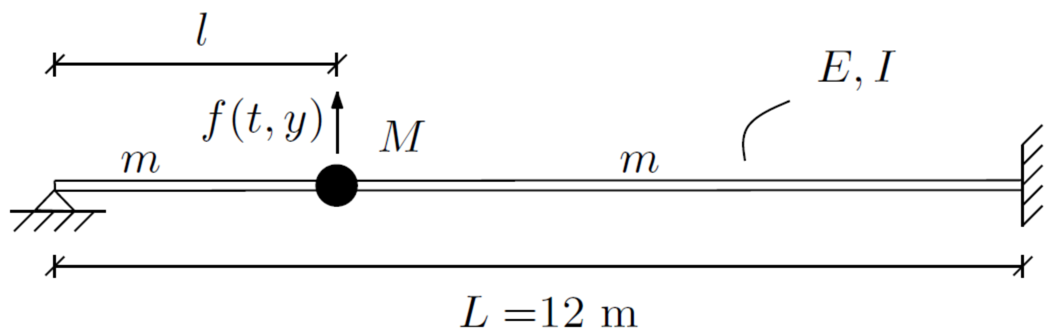


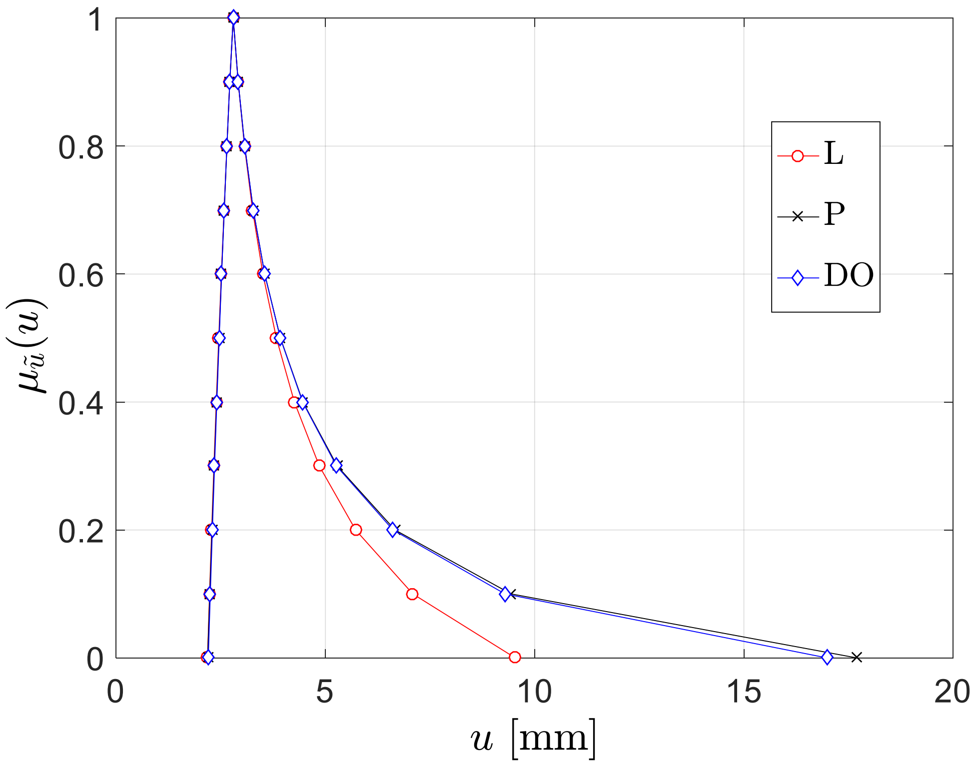


*Beer, M. & Liebscher, M. Designing robust structures - A nonlinear simulation based approach. Computers & Structures, 2008, 86, 1102-1122

Fuzzy Analysis: Linear Dynamics

- <u>Example 1</u>
  - Steel beam subject to harmonic loading*
  - Distributed and point mass*

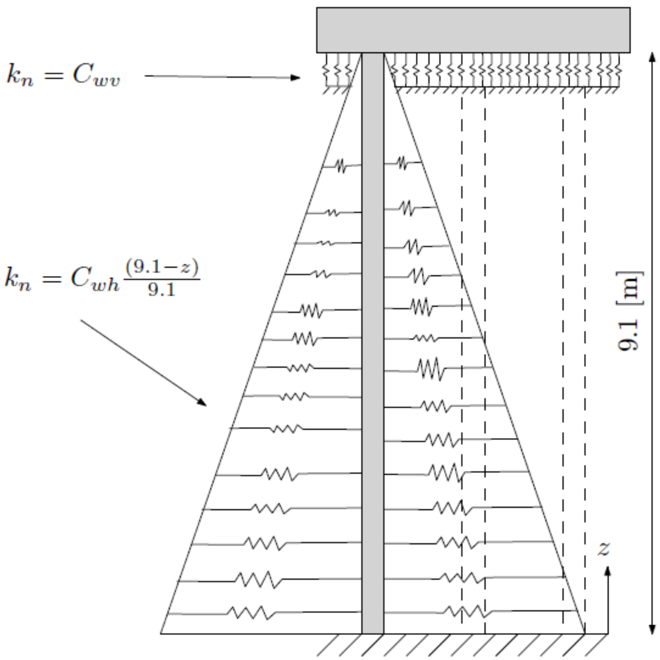


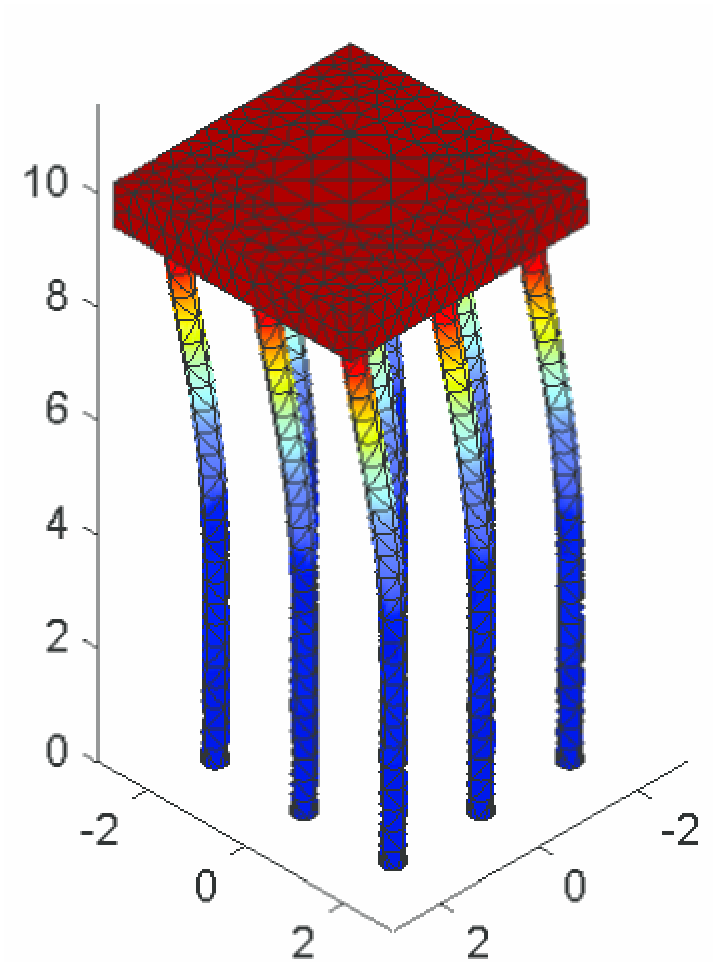


Appr.	Description
L	FRF approximated considering linear expansion of spectral properties
P	FRF approximated considering intervening variables for approximating frequencies
DO	Exact FRF

Fuzzy Analysis: Linear Dynamics

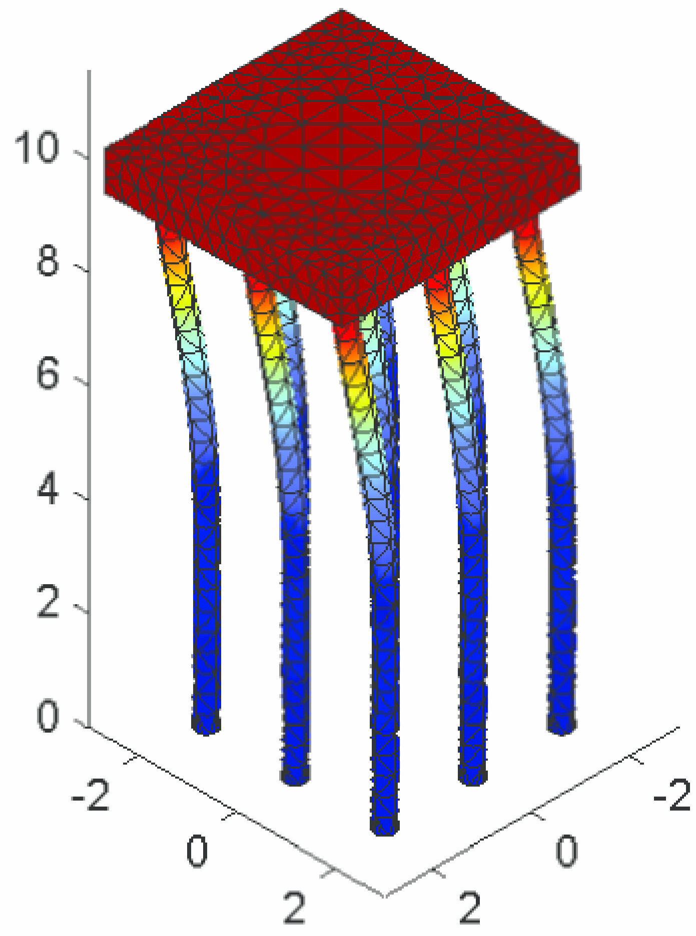
- Example 2
  - Concrete slab over foundation piles
  - Load: rotatory machine over slab
  - Soil: Winkler model
  - Model: 27000 DOF





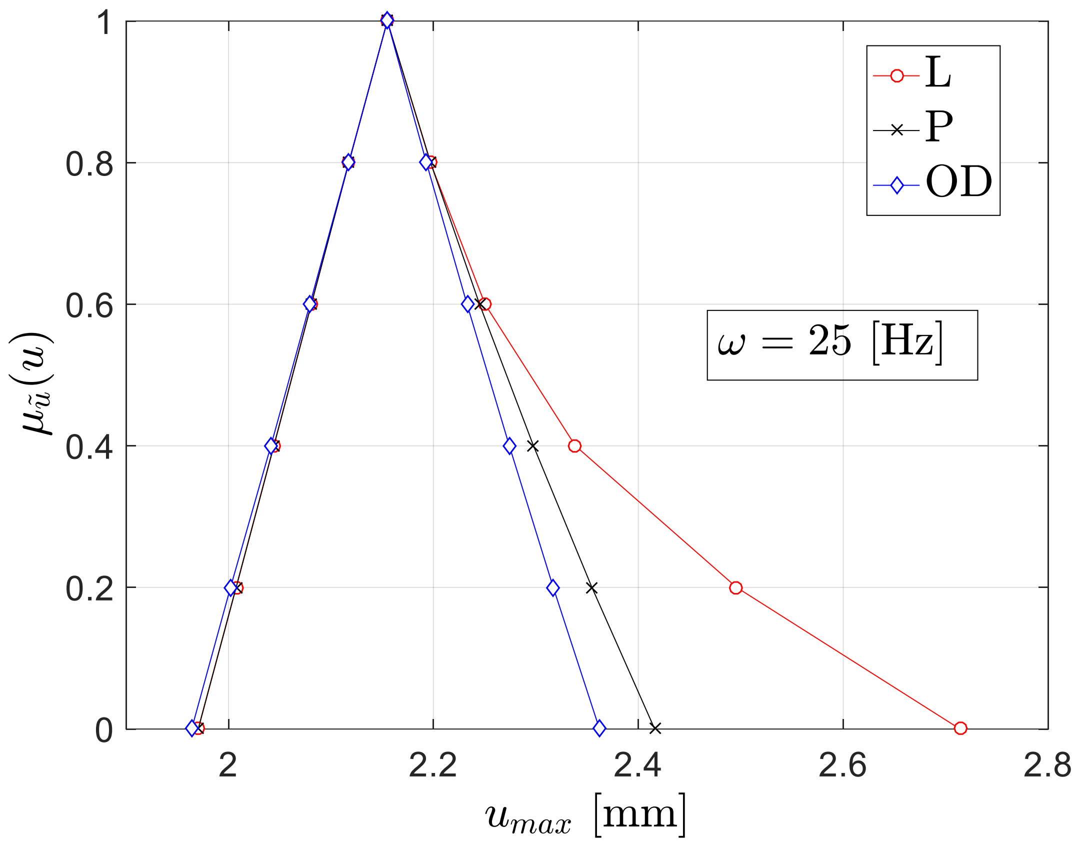
Fuzzy Analysis: Linear Dynamics

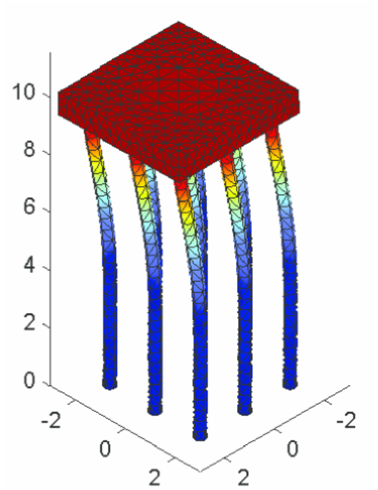
- <u>Example 2</u>
  - Concrete slab over foundation piles
  - Fuzzy parameters: soil properties, elastic properties, damping, loading
  - <u>Response of interest</u>: displacements of the slab, central node for a specific frequency



Fuzzy Analysis: Linear Dynamics

- <u>Example 4</u>
  - Concrete slab over foundation piles





Appr.	Description
L	FRF approximated considering linear expansion of spectral properties
P	FRF approximated considering intervening variables for approximating frequencies
DO	Exact FRF

Conclusions & Outlook

- Intervening variables: highly accurate approximations for natural frequencies (dynamics)
- Intermediate quantities: simple approach, allows to capture nonlinearity
- Sensitivity analysis provides valuable information for approximating structural response
- Proposed framework allows performing fuzzy structural analysis with ***reduced numerical costs***

- Future research efforts
  - Improve quality of estimates for response membership functions
  - Alternative models to improve quality of approximations
  - Extend methodologies to more general cases (e.g. fuzzy analysis applied to nonlinear structures)