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STRUCTURAL CONDITION ASSESSMENT USING IMPRECISE PROBABILITY

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Reliability of Structures

In structural engineering, the reliability and safety of a structure must be accurately assessed.

To achieve this reliability, the uncertainties present in both the structure and applied loads must be included in the analytical schemes.

Attributions of Uncertainty

- Load variations obtained from historical records and/or through measurements in the field or laboratory
- Numerical modeling errors caused by oversimplification of the system and the boundary conditions inaccuracies
- Variations in material properties
- Manufacturing errors in structural member dimensions
- Limited test data

Treating Uncertainty

Although theories of structural reliability are well-established, the practical application of the methods developed for reliability analysis is mathematically complicated.

Mathematical complexity increases dramatically as the number of components or modes of failure increases.

As a result, practicing engineers often resort to gross simplifications to overcome the complexity inherent in the general formulation of structural reliability.

This leads to reliability predictions that have a significant levels of error.

Isomorphic Uncertainty Categorization

- Aleatoric

The system has an intrinsic random or stochastic nature and it is not predictable. It cannot be reduced by higher precision modeling or additional information and requires random variables to follow assumed distributions.

- Epistemic

The uncertainty induced by the lack of information and it is predictable. It is unable to provide likelihood of parameter values within the parameter's bounds.

Imprecise Probability

Imprecise probability is a polymorphic approach of quantifying uncertainty that provides a framework that addresses the shortcomings of both probabilistic and possibilistic methods by considering possibilistic bounds on probabilistic models.

Research Objective

To develop a method for reliability analysis of a structure using imprecise probability approach to attain a more realistic yet simpler process of treating uncertainties than traditional probabilistic-based reliability analyses.

Presentation Outline

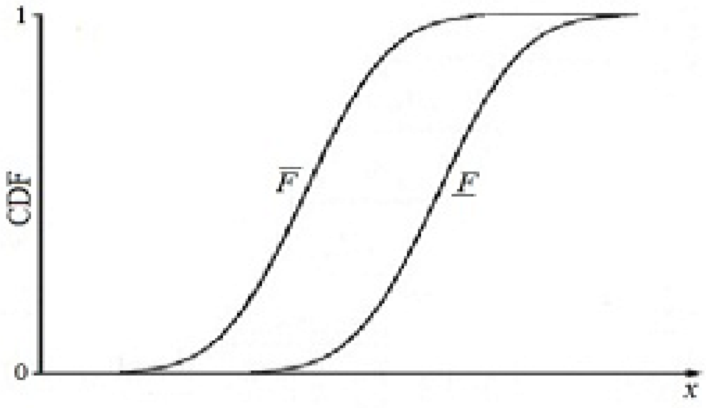
- Review of Imprecise Probability Structures
- Review of Probability Based Structural Reliability Analysis
- Introducing the method of Imprecise Probability Structural Condition Assessment
- Case Study
- Conclusions

Imprecise Probability

Imprecise probability is a polymorphic approach for quantifying uncertainty.

It allows for setting possibilistic bounds on probabilistic models.

Therefore, it requires fewer assumptions which may significantly influence the results of the analysis.



Traditional Probabilistic Reliability Analysis

Considering a performance function, Z, with multiple independent variables representing the design parameters, X_i :

$$Z = g(X_1, X_2, \ldots, X_n)$$

Using performance limit z_o, the probability of failure is:

$$P_f = P(Z \leq z_o)$$

Similarly,

$$P_f = \int_{-\infty}^{z_o} f_z(z)dz$$

where, $f_z(z)$ is the probability density function of the performance function in the multivariate space.

Traditional Probabilistic Reliability Analysis (Cont.)

Generally, there are multiple modes of failure present.

Consideration of the two extreme cases of independence and perfect correlation among the modes allows for setting bounds on the probability of failure for the structure as:

$$\max(P_f^1, P_f^2, \ldots, P_f^m) \leq P_f \leq 1 - \prod_{j=1}^{m} (1 - P_f^j)$$

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Imprecise Probability Structural Condition Assessment

General algorithm for Imprecise Probability Structural Condition Assessment:

I. Determine the structure’s modes of failure (e.g. bending, shear, deflection).

II. Determine the imprecise probability structure for the performance function for each failure mode.

Imprecise Probability Performance Function

For each mode of failure:

a. Construct independent imprecise probability structures for the uncertain load and resistance.

b. Perform random sampling on the CDF probability levels of uncertain load and resistance imprecise probability structures.

Imprecise Probability Performance Function (Cont.)

For each realization *r* of the simulation:

- Randomly select independent CDF values for load and resistance constructed imprecise probability structures and compute the corresponding interval load $\tilde{S}$ and interval resistance $\tilde{R}$.

- Determine and store interval bounds on the uncertain performance function.

c. Repeat sufficiently large number of realizations to construct imprecise probability structure for uncertain performance function.

Imprecise Probability Structural Condition Assessment (Cont.)

III. Determine the interval probability of failure for each failure mode by computing the performance function at the performance limit for each bound of the corresponding imprecise probability structure.

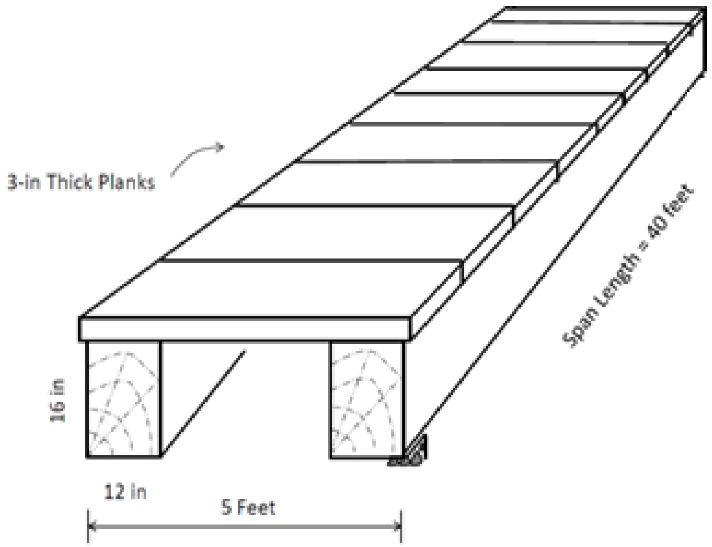
IV. Determine the interval probability of failure of the structure for two extreme cases of perfect correlation and independence among the modes as:

$$\max\left[\min(P_f^1), \min(P_f^2), \ldots, \min(P_f^m)\right] \le P_f \le 1 - \prod_{j=1}^{m}\left[1 - \max(P_f^j)\right]$$

V. Determine the maximum probability of failure as the upper bound of the interval probability of failure of the structure (independence case).

Case Study

Investigating the structural condition of a timber pedestrian bridge.



There is an increase in the bridge’s live load (pedestrians using the bridge) which has raised a concern over its safety.

Problem Parameters

The dominant modes of failure include bending, shear, and deflection modes of failure for the two beams. The probabilistic values (Gaussian distributions) are:

Mode	**Resistance**		**Load**	
Bending	*Bending Capacity*		*Applied Bending Stress*	
	μ_R^b (psi)	σ_R^b (psi)	μ_S^b (psi)	σ_S^b (psi)
	6,990	1,820	1,730	416
Shear	*Shear Capacity*		*Applied Shear Stress*	
	μ_R^s (psi)	σ_R^s (psi)	μ_S^s (psi)	σ_S^s (psi)
	825	214	57.8	13.9
Deflection	*Deflection Limit*		*Induced Deflection*	
	μ_R^d (in)	σ_R^d (in)	μ_S^d (in)	σ_S^d (in)
	4.0	0	2.41	1.01

(Mohammadi and Modares 2013)

Traditional Probability Analysis

Consider a performance function and performance limit

$$Z = R - S$$

$$z_o = 0$$

Linearization of the performance function (First Order Reliability Method, FORM) and using a Gaussian distribution, the failure probability of each mode is obtained as :

$$p_F = \Phi\left(\frac{z_0 - \mu_Z}{\sigma_Z}\right)$$

or:

$$p_F = \Phi\left(\frac{\mu_S - \mu_R}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right)$$

where, Φ is the standard normal cumulative distribution function.

Traditional Probability Analysis (Cont.)

The probability of failure for each failure mode

Mode	**Probability of Failure P_f**
Bending	2.40×10^{-3}
Shear	1.73×10^{-4}
Deflection	5.77×10^{-2}

Considering the two extreme cases of perfect correlation and independence among the failure modes, the probability of failure for the structure:

$$0.0577 \le P_f \le 0.0602$$

To ascertain the reliability of the structure, the upper bound (independence case) can be used for the reliability level of the structure, $P_f = 0.0602$

Traditional Probability Analysis (Cont.)

- Traditional probability analysis methods (including FORM) are not capable of considering uncertainties and variations in the mean or standard deviation of either load or resistance.

- This is a major shortcoming of those methods that practicing engineers have encountered.

- The framework of imprecise probability structures allows for consideration of these uncertainties .

Imprecise Probability Condition Assessment

- Resistance and load values defined by imprecise probability structures for all failure modes.

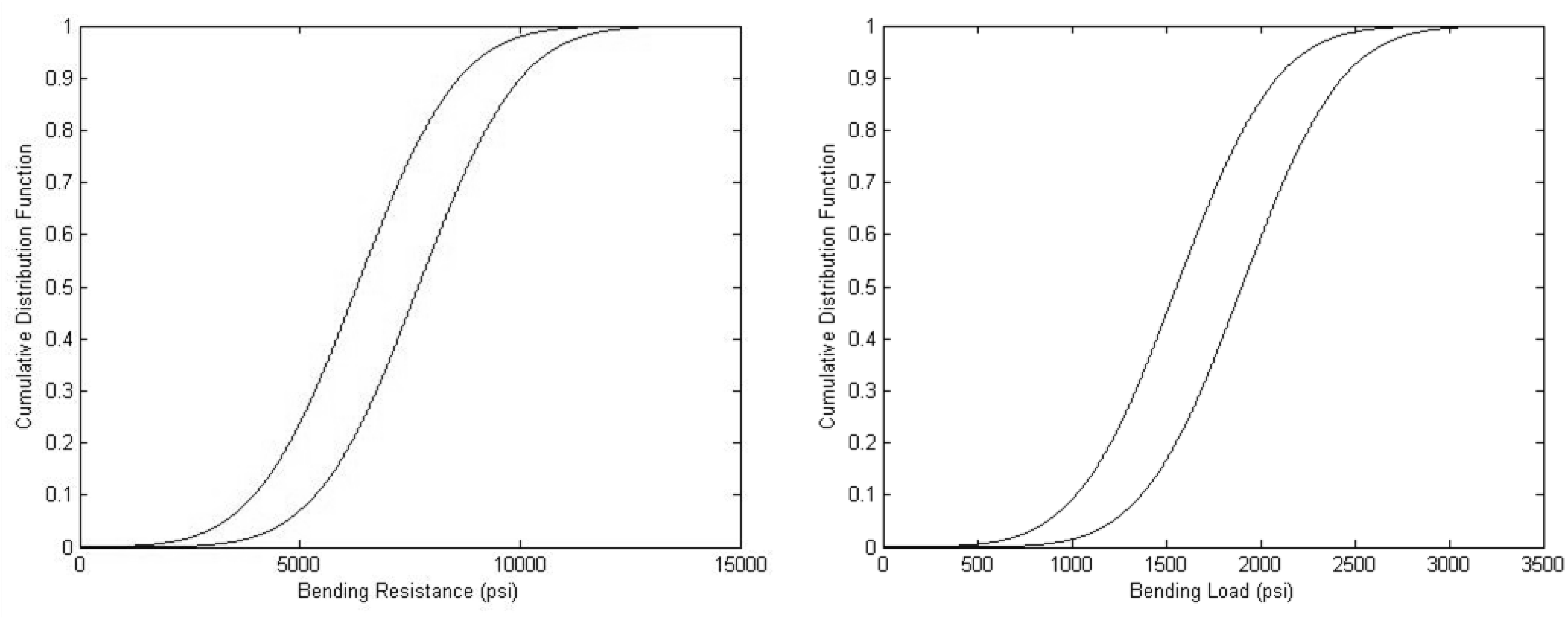
- All imprecise probability structures were generated by considering a ±10% shift in the mean values.

- As the resistance for the deflection mode of failure is a code limit, it is not modeled as an imprecise probability structure.

- The analysis is performed for each mode of failure as well as the entire structure.

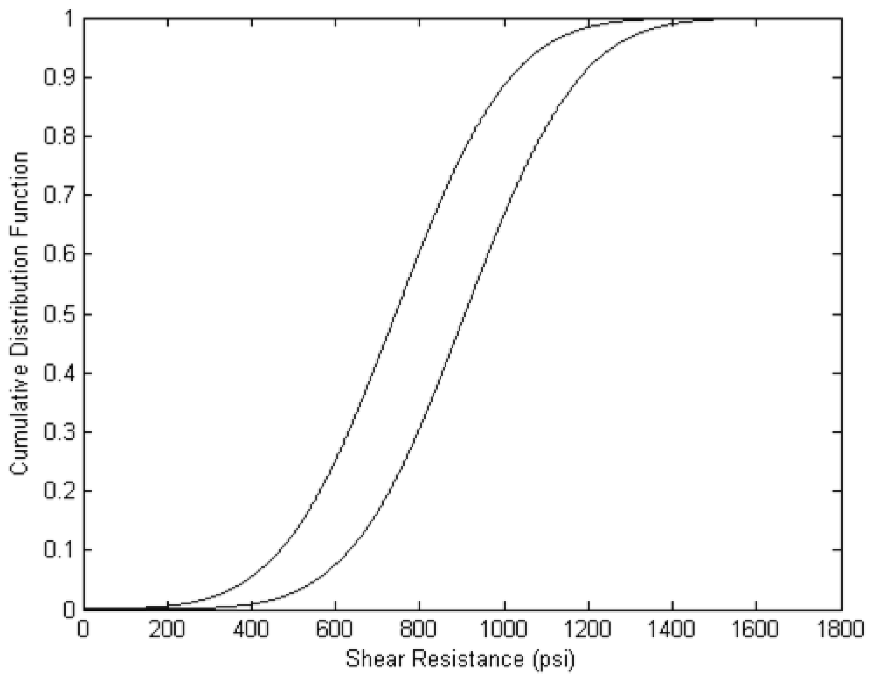
Bending Mode: Imprecise Probability Structures

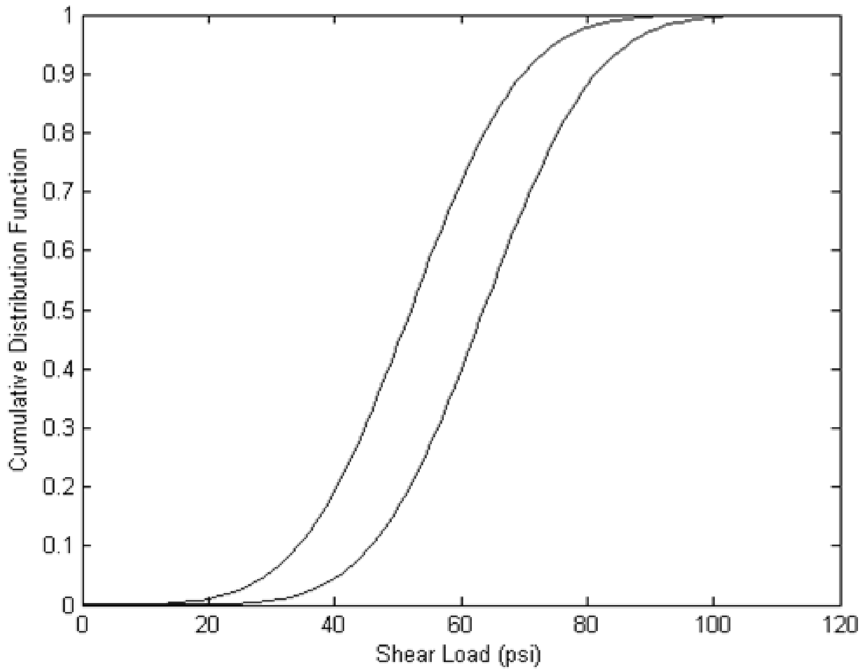
Imprecise probability structures for the resistance and load in bending mode



Shear Mode: Imprecise Probability Structures

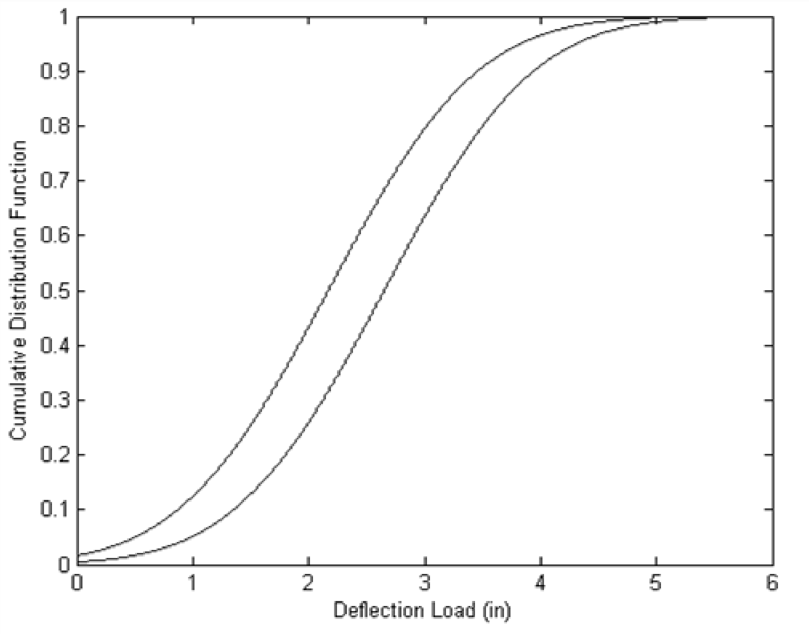
Imprecise probability structures for the resistance and load in shear mode





Deflection Mode: Imprecise Probability Structure

Imprecise probability structure for the induced deflection in deflection mode



Results - Modes of Failure

Interval bounds on the probability of failure for each failure mode
(10^6 Monte-Carlo Simulations)

Bending	
Lower Bound	*Upper Bound*
4.76e-04	9.36e-03

Shear	
Lower Bound	*Upper Bound*
3.00e-05	6.98e-04

Deflection	
Lower Bound	*Upper Bound*
3.50e-02	9.12e-02

The probability of failure of the structure is dominated by the single failure mode with the greatest probability of failure.

Results - Structure

Interval bounds on the probability of failure for structure
(10^6 Monte-Carlo Simulations)

Present Method	
Lower Bound	*Upper Bound*
3.50e-02	1.00e-01

FORM	
Lower Bound	*Upper Bound*
5.77e-02	6.02-02

The results determined using the present method contain the FORM results, verifying the developed method.

Conclusions

- A new method for reliability analysis of a structure using an imprecise probability approach is developed.

- This method offers a new direction for incorporating uncertainties in condition assessment of structural systems.

- It allows for independent uncertainty in the load and resistance for each mode of failure using imprecise probability structures.

- As this method does not place restrictive assumptions typical in traditional probabilistic structural condition assessment schemes, it provides a more comprehensive process of treating uncertainties than those schemes.

- The simplicity of the proposed method makes it attractive for introducing uncertainty defined by imprecise probability into structural condition assessment procedures.

Thank you