BAYESIAN CALIBRATION OF LATTICE DISCRETE PARTICLE MODEL FOR CONCRETE

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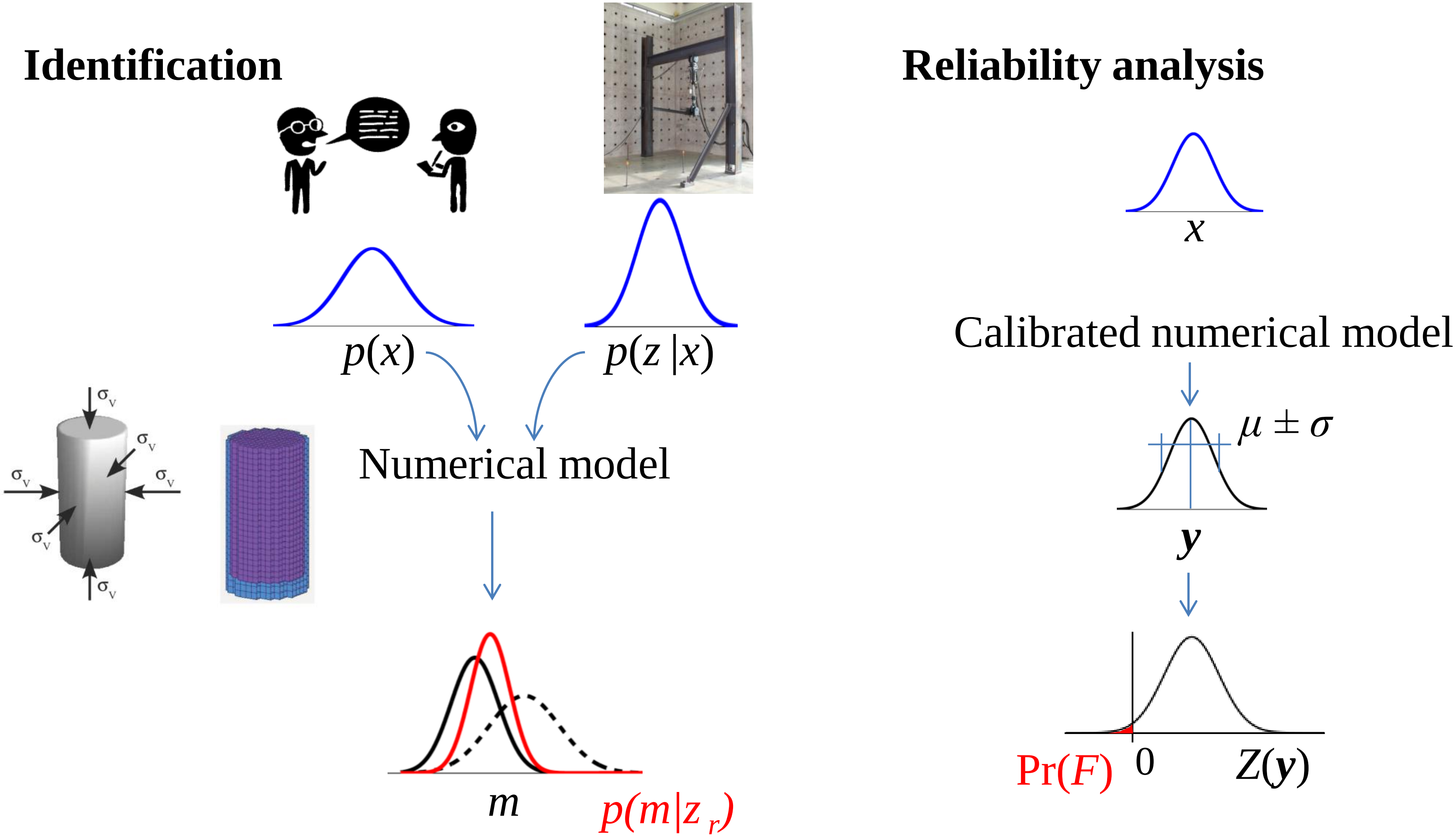
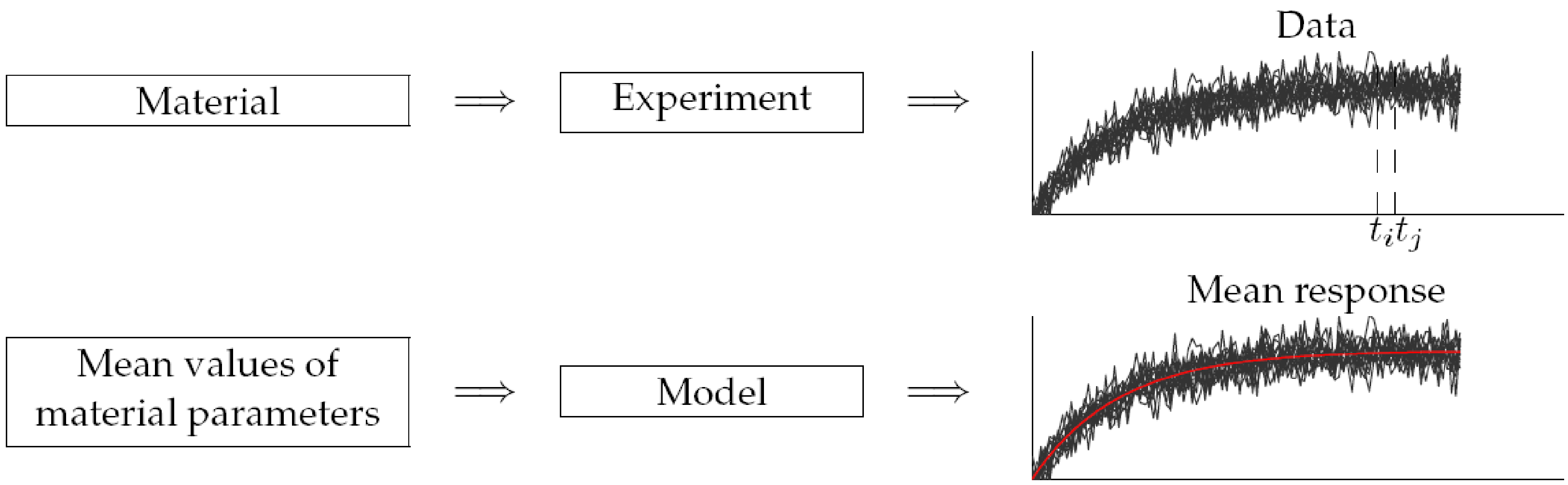
- Reducing uncertainties in input parameters
- Investigation of uncertainties in structural reliability analysis



Fitting the model response to experimental data

- The most common approach of parameter estimation
- Parameter optimisation (ill-posed problem) – robust optimisation algorithms



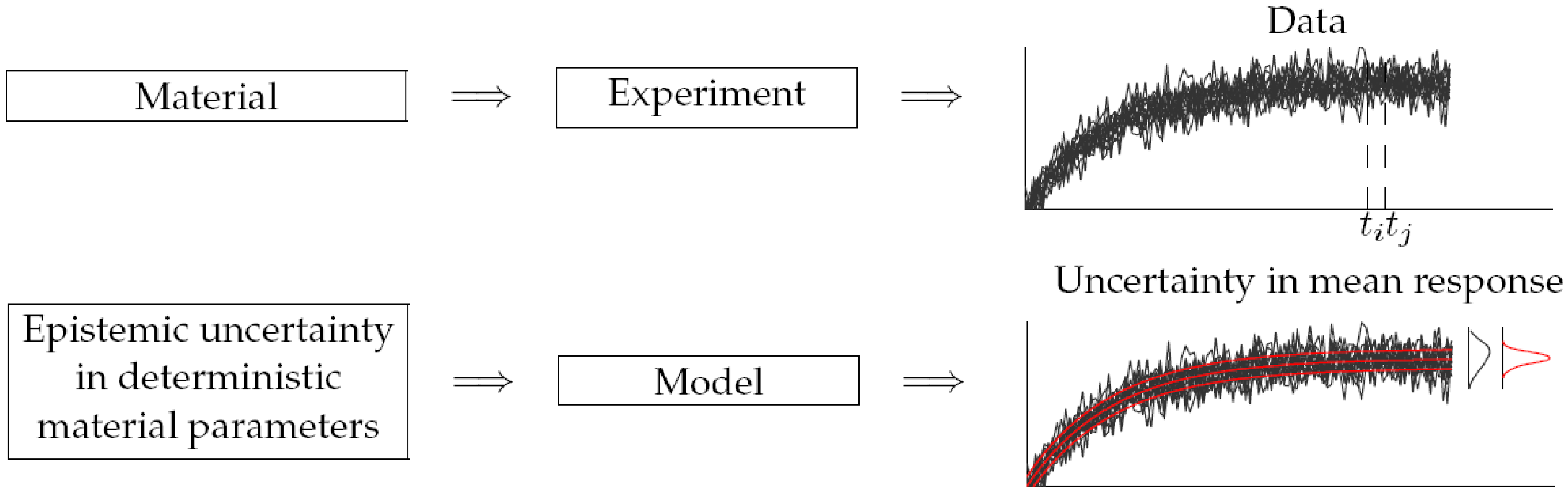
Probabilistic parameter identification

Quantification of epistemic uncertainties

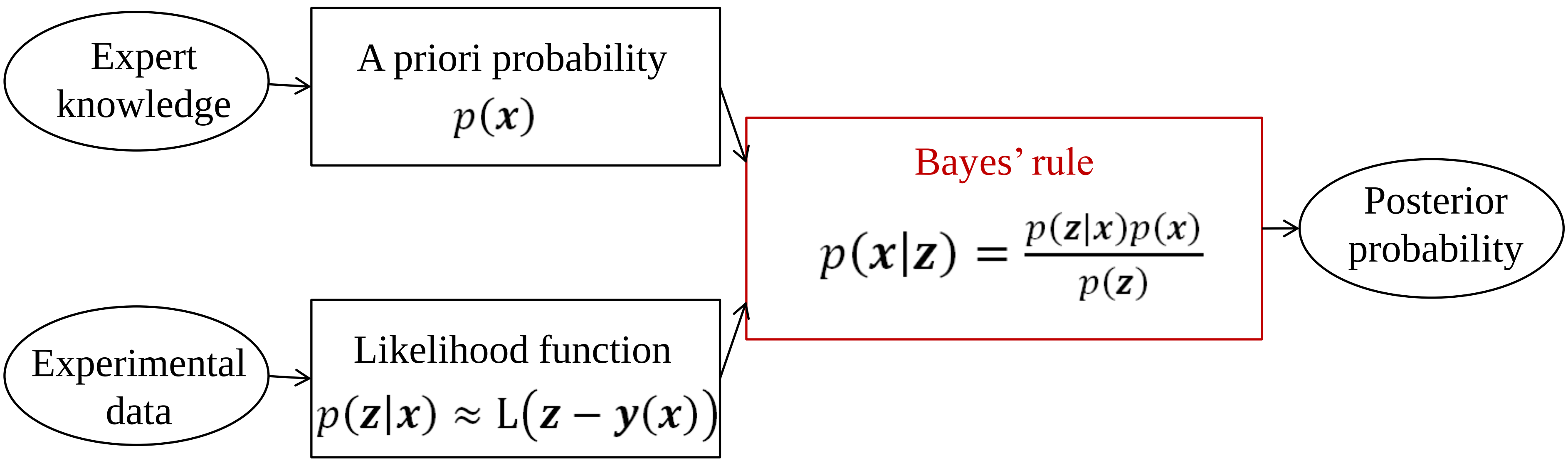
– Epistemic (reducible, subjective) uncertainty about deterministic values

– Bayesian approach combining all available information

– Well-posed identification problem



Setting of Bayesian approach



$$\boldsymbol{z}(\boldsymbol{x}, \boldsymbol{\omega}) = \boldsymbol{y}(\boldsymbol{x}) + \varepsilon(\boldsymbol{\omega})$$

Data
Random observations

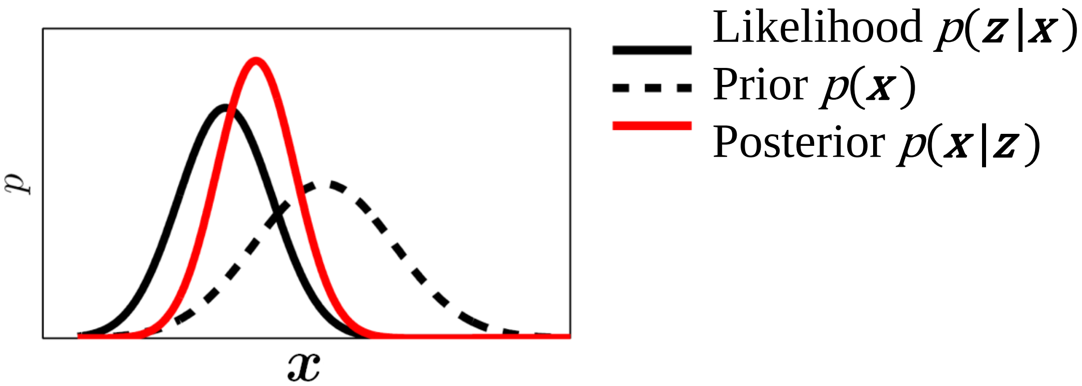
Model response
Often a black-box function
Expensive to evaluate

Measurement error
Usually $\varepsilon \sim N(0,\Sigma)$

$\boldsymbol{x}$ – model parameters to be identified

Methods of computing Bayesian posterior

$$p(\boldsymbol{x}|\boldsymbol{z}) = \frac{p(\mathbf{z}|x)p(x)}{p(\mathbf{z})} = \frac{p(\mathbf{z}|x)p(x)}{\int p(\mathbf{z}|x)p(x)dx} \approx \mathrm{L}\big(\boldsymbol{z} - \boldsymbol{y}(\boldsymbol{x})\big)\ p(\boldsymbol{x})$$

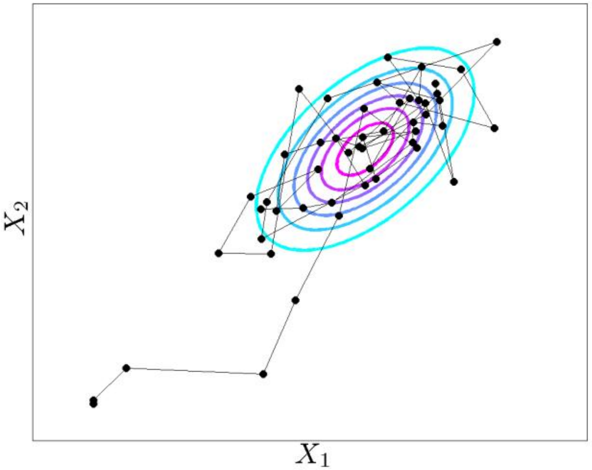


- Markov chain Monte Carlo
- Kalman filter
- Optimal maps

Markov Chain Monte Carlo (MCMC)

Features of the method

- Sampling procedure based on model simulations, suitable for nonlinear models
- Markov chain of required stationary distribution equal to the posterior
- Different algorithms:
  - Gibbs sampler
  - Metropolis-Hasting algorithm
  - Metropolis algorithm



Disadvantages

- Low convergence of the method ⟶ appropriate setting of the algorithm:
  - Proposal distribution
  - Starting point / burn-in period
- High computational effort ⟶ Model approximation:
  - Neural network, radial basis function, kriging, polynomial chaos expansion

Polynomial chaos expansion (PCE)

Approximation of a model response

$$\widetilde{PC}(\boldsymbol{x}(\boldsymbol{\xi})) = \sum_{\alpha} \boldsymbol{\beta}_{\alpha} \psi_{\alpha}(\boldsymbol{\xi})$$

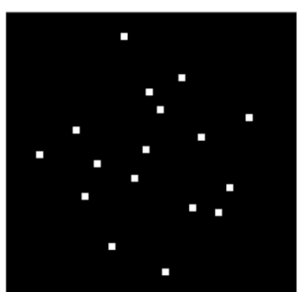
- Respect to probability distribution of random variables

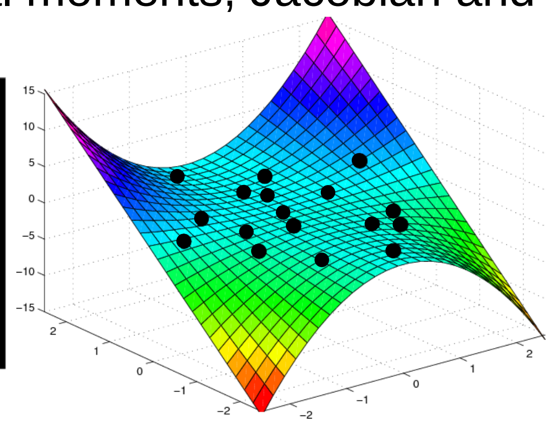
  → Hermite polynomials – Gaussian, Legendre polynomials – Uniform

- Methods for construction of PCE-based approximation

  → Linear regression, stochastic Galerkin method, stochastic collocation method

- Efficient evaluation of statistical moments, Jacobian and Sobol' sensitivity indices



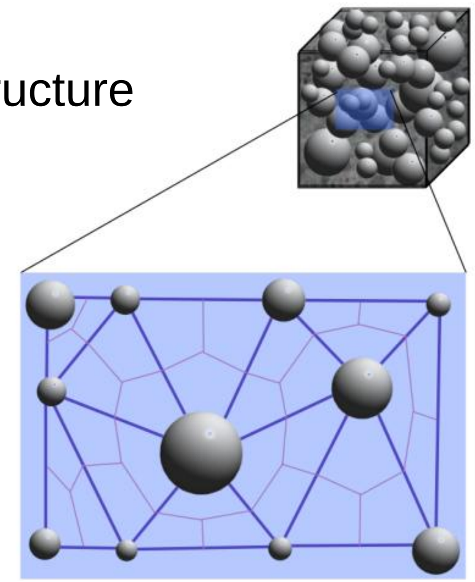


[Cusatis, 2011]

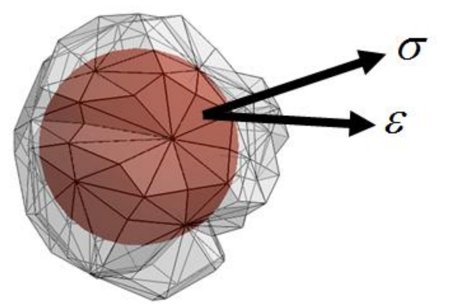
Lattice discrete particle model for concrete

- A priori volume discretization is performed taking into account material heterogeneity (coarse aggregate pieces)
- Parameters governing the generation of concrete meso-structure

material property		unit	value
minimum particle size	d_0	mm	4
maximum particle size	d_a	mm	16
cement content	c	kg/m^3	240
water to cement ratio	w/c	-	0.83
aggregate to cement	a/c	-	8.83
Fuller coefficient	n_F	-	0.5
concrete density	ρ	kg/m^3	2400



- Randomly generated concrete granulometric distribution acts as a noise in the model response



Bayesian model calibration

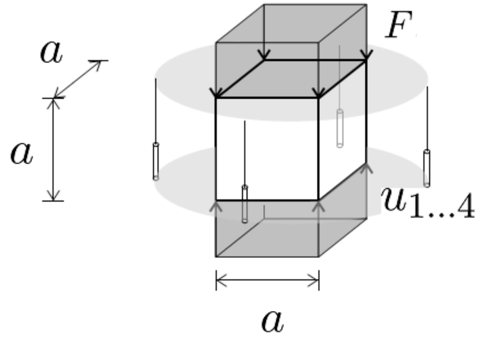
Parameters to be identified and their prior distribution

material property		unit	value (range)
normal modulus	E_0	MPa	$20000 - 70000$
shear-normal coupling	α	-	$0.2 - 0.3$
tensile strength	σ_t	MPa	$1.5 - 5$
tensile characteristic length	l_t	mm	$50 - 300$
softening exponent	n_t	-	$0.1 - 1$
shear/strength ratio	σ_s/σ_t	-	$1.5 - 8$
initial friction	μ_0	-	$0.001 - 0.5$
compressive strength	σ_{c0}	MPa	$\sigma_{c0} = 40\sigma_t$
transitional stress	σ_{N0}	MPa	$\sigma_{N0} = 240\sigma_t$

Experimental data

- **Uniaxial compression test**
  - 3 repetitions
  - Nominal stress discretised into 250 strain steps

$$\sigma_N = \frac{F}{a^2} \quad \text{and} \quad \varepsilon_N = \frac{u}{a}$$

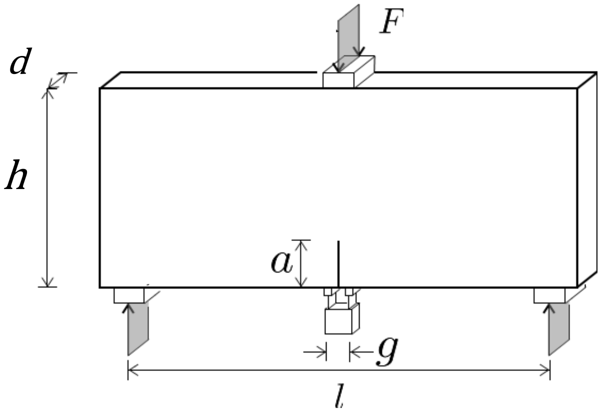


- **Notched three-point-bending test**
  - 4 repetitions
  - Nominal stress discretised into 250 strain steps

$$\sigma_N = \frac{3Fl}{dh^2} \quad \text{and} \quad \varepsilon_N = \frac{CMOD}{h}$$

  - Elastic stiffness K

$$\varepsilon_N^{\text{inel}} = \varepsilon_N - \sigma_N\,(1/K)$$



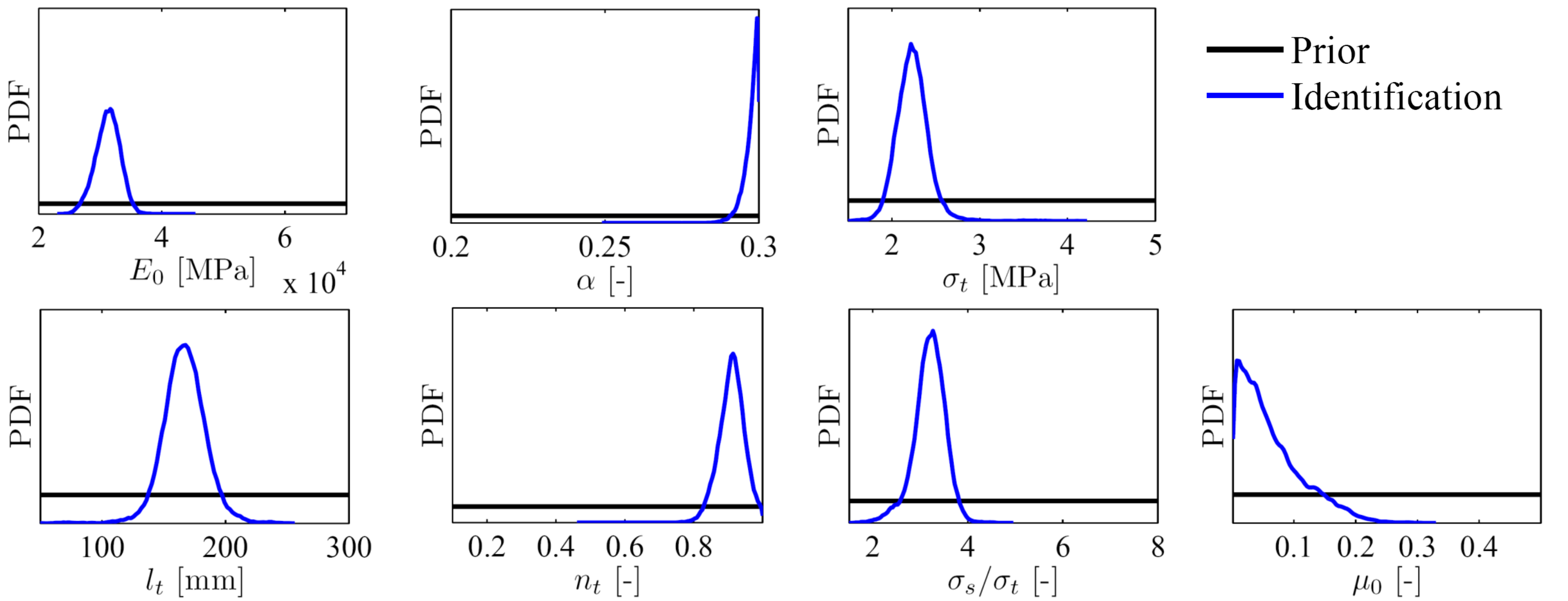
Bayesian model calibration

Bayesian posterior estimate

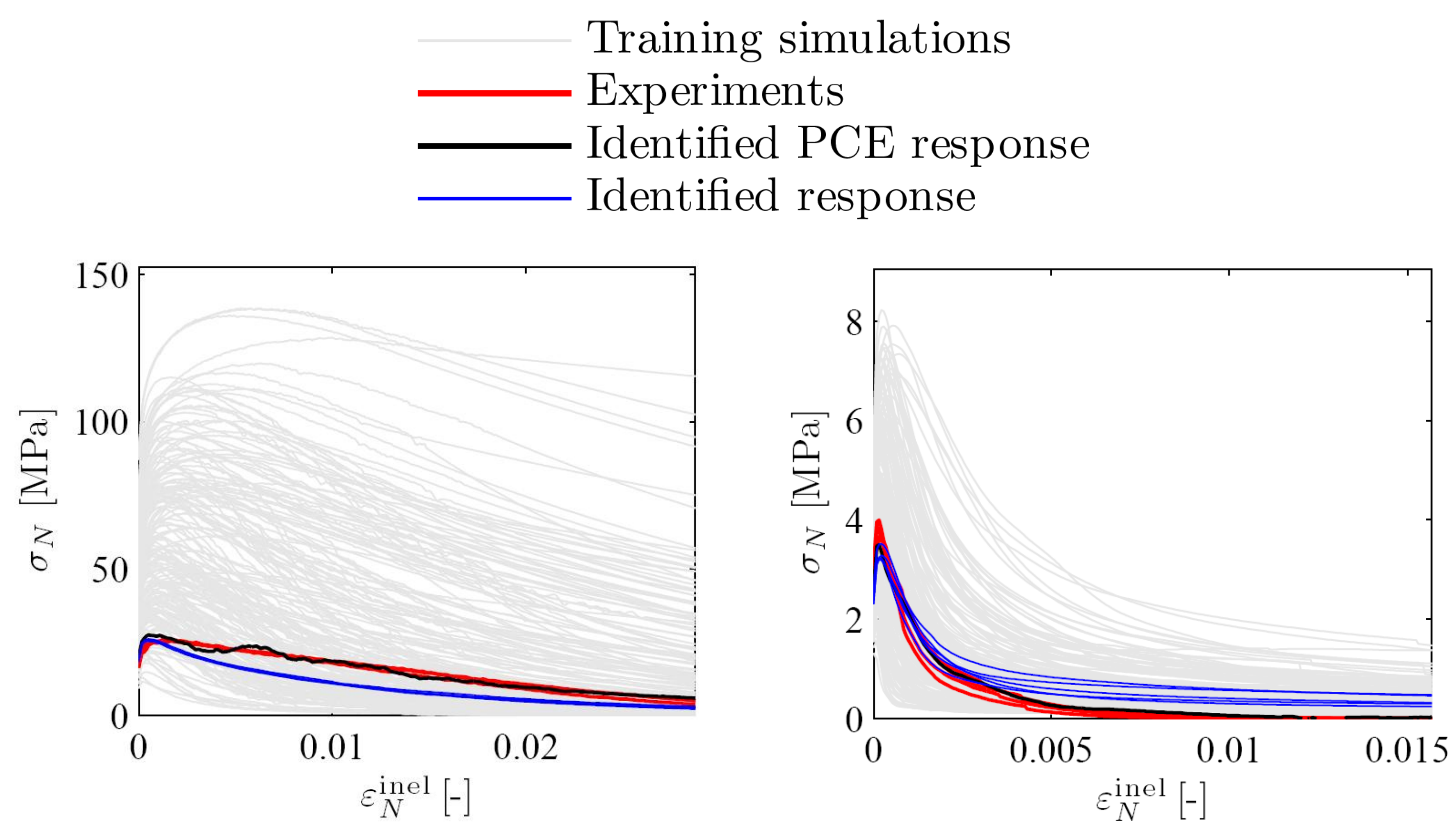
- Uniform prior distribution
- Normal distribution of experimental errors
  - Compression test: $\varepsilon_{\sigma_N} \sim N(0, 8^2)$
  - Three-point bending test: $\varepsilon_{\sigma_N} \sim N(0, 2^2)$, $\varepsilon_K \sim N(0, 2880^2)$
- MCMC sampling
  - Metropolis algorithm, 500,000 samples
- PCE-based model approximation
  - Legendre polynomials of third degree
  - Linear regression based on 200 full model simulations
  - Elimination of the noise caused by random distribution of particles

Identified parameters' PDF

	E_0 [MPa]	α [-]	σ_t [MPa]	l_t [mm]	n_t [-]	σ_s/σ_t [-]	μ_0 [-]
MEAN	31183	0.297	2.236	166.6	0.910	3.192	0.063
STD	1998	0.003	0.193	17.8	0.037	0.338	0.051



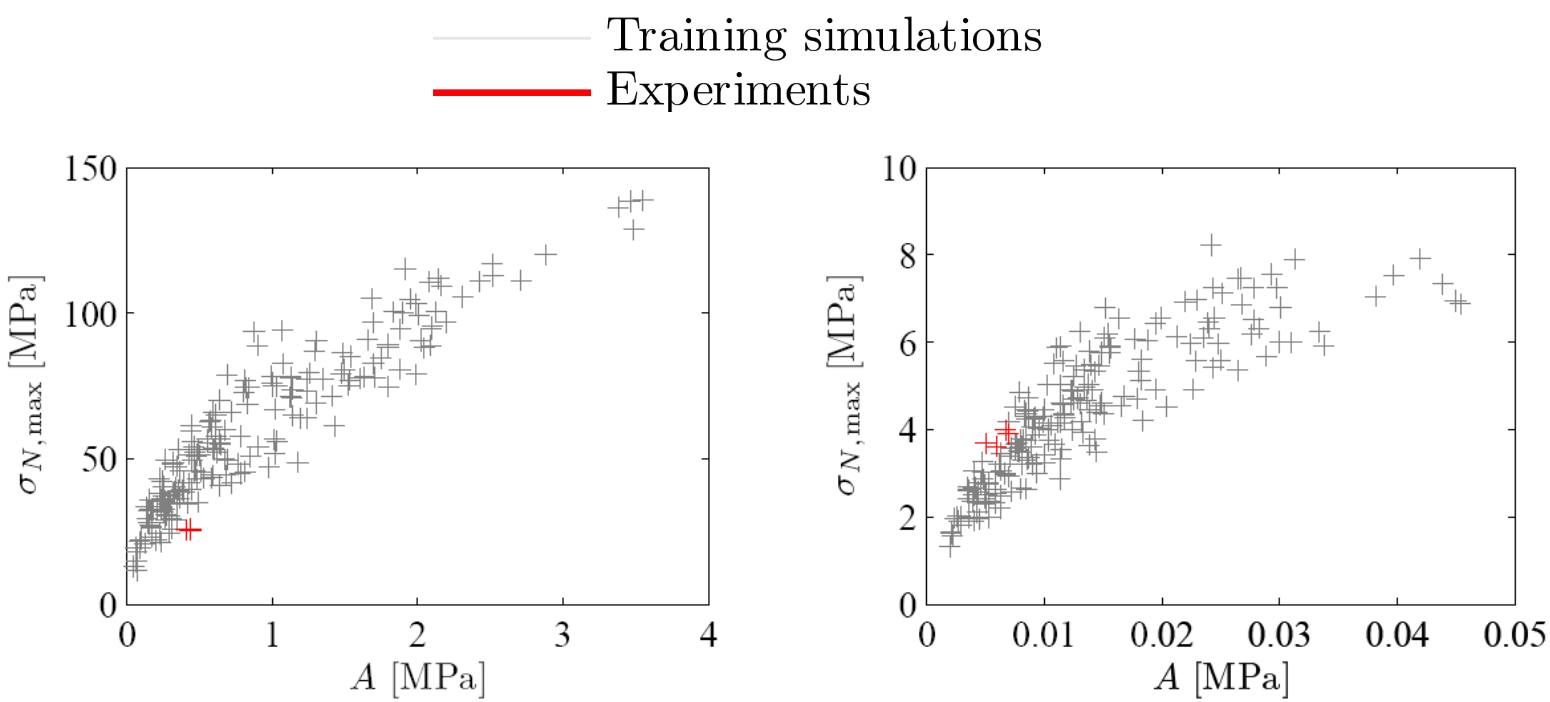
Comparison of model response and experimental data



Uniaxial compression test

Three-point-bending test

Inappropriate choice of prior bounds



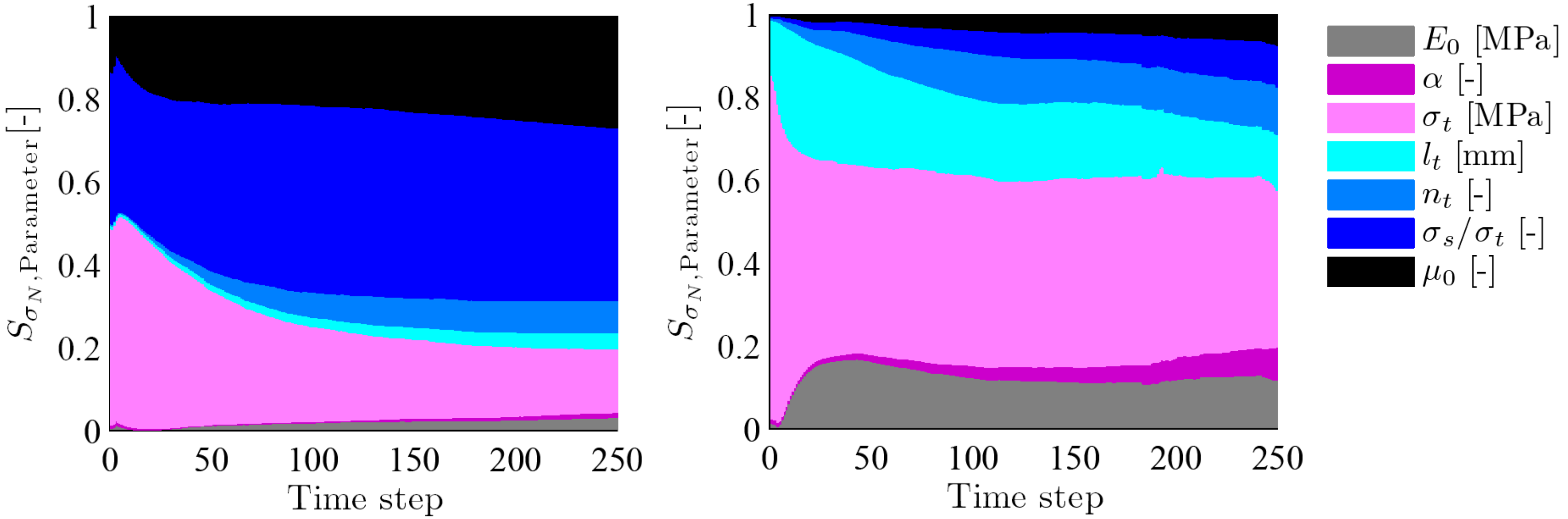
Uniaxial compression test

Three-point-bending test

$$A = \int \sigma_N \mathrm{d}\varepsilon_N$$

Sensitivity analysis

Uniaxial compression test | Three-point-bending test



Conclusion

Bayesian model calibration

- **Combination of prior knowledge and noisy experimental observations**
  - Estimation of unknown model parameters
  - Probabilistic description of epistemic uncertainty in deterministic values
- **MCMC – Sampling procedure**
  - Versatile, model-independent, computationally exhaustive method
- **Polynomial chaos-based approximation**
  - Acceleration of identification procedure, sensitivity analysis
- **Calibration of lattice discrete particle model**
  - Inaccurate approximation in the region of experimental data caused by inappropriate choice of prior distribution
  - Prescribtion of a new prior ranges to obtain the necessary information for constructing the accurate model approximation

THANK YOU FOR YOUR ATTENTION.

Aknowledgements

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