Inverse Analysis of Coupled Hydro-Mechanical Problem in Dynamically Excited Dams

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Abstract: Dams play a major role in the development and sustenance of communities, economies and civilizations. Apart from being exposed to a variety of dynamic forces (ambient and induced), many dams today have been in operation beyond their design life cycle and thus uncertainty and deterioration in the material properties would be expected. In order to reduce the risk of failure of these structure, a strict and frequent monitoring of the structure is required. However as a result of the physical scale of most dams, this is cumbersome and in many cases expensive. In an attempt to gain insight into the physical properties (such as stiffness and density distribution, permeability, etc) which are crucial to the effective operation of these dams an inverse analysis is carried out on the coupled hydro-mechanical finite element model. The parameter identification is done using experimental data obtained from recorded responses resulting from induced vibrations on the dams' crest. As with most engineering problems, we are faced with an ill-posed problem. Thus we intend to propose an efficient algorithm for the inverse problem and also ascertain the quality of the calculated parameters by making a comparison between the simulated and recorded dam response.

Keywords: inverse problem, optimization, structural health monitoring

1. Introduction

Dams are structures used to retain or control the flow of water in a catchment area, this could be for agricultural purposes (irrigation), flood prevention or for electricity generation. Although the demand for such structures are on the increase to cater for the increasing population and demand for renewable energy, building new dams requires lots of time and money, thus the efficiency and operating capacities of existing dams should be maximized.

Today most dams have been in operation far beyond their designed operation period and as such a decrease in the reliability of such structures is expected. As much as these structures are of great value to communities, a failure in such a structure is in most cases catastrophic, thus great care must be taken in its design, operation and maintenance. In order to assess both the structural integrity and reliability of these structures routine inspections are required. However as a result of the physical sizes and the increasing heterogeneity in the material properties of these structures it is often very difficult and tedious to carry out both a qualitative and quantitative assessment of the dams.

A number of literature proposing models based on a variety of formulation for the analysis of various phenomena during the design and operation of a dam exist. These includes (Lahmer, 2010), (Lahmer et al., 2011), and (Khoei et al., 2014).
Furthermore, in order to ascertain the structural integrity and monitor the performance of these very important structures irrespective of when it was constructed, it is necessary to obtain reliable values of certain parameters which are crucial to the safe operation of these dams. These parameters provide vital information such as material heterogeneity, saturation distribution, crack or void location, stiffness degradation etc. The identification of the parameters is made possible by the inverse analysis of a validated numerical model that adequately describes the dams' behavior during its life cycle.

2. Mathematical Formulation

The finite element methods are employed in the calculations of the displacements, pore pressures, stresses and strains induced on the dynamically excited dam. The mechanical system involves the solution of the second order differential equation, derived from Newtons' equation of motion. Where the time varying excitation force is $\{f_t\}$, the mass, damping and stiffness are denoted by $[M]$, $[C]$ and $[K]$, the displacement, velocity and accelerations are also denoted by $\{u\}$, $\{\dot{u}\}$ and $\{\ddot{u}\}$ in Eq. (1). The mass matrix is formulated for each element as obtained in Eq. (2), the stiffness in Eq. (3) and the resulting damping matrix is obtained in Eq. (4).

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \left\{f_{(t)}\right\} \tag{1}$$

$$m^e = \int_\Omega N^{e^T} \rho^e N^e d\Omega \tag{2}$$

$$k^e = \int_\Omega B^{e^T} D^e B^e d\Omega \tag{3}$$

where:
N^e is the element shape function
ρ^e is the density of the element
B is a matrix containing the derivatives of the shape function
D is a stress/strain matrix which is a function of the materials Youngs' modulus, E and Poisson's ratio, ν

$$[C] = \alpha [M] + \beta [K] \tag{4}$$

The α and β coefficients in Eq. (4) are the Rayleigh damping coefficients. These can be obtained from experiments or through other methods proposed in literature. The method proposed by (Chowdhury and Dasgupta, 2003) was used to calculate α and β.

The Newmark scheme (Chopra, 1995) is applied in solving Eq. (1) such that the values of the nodal displacements, velocities and accelerations are obtained. Depending on the β^* and γ^* values used to implement the scheme, it is necessary that the time step δt chosen is less than the critical time step T_c to ensure stability. The resulting displacements obtained are used to compute the strains and stresses as done in standard FEM calculations.

In order to get a more realistic idea of the dams' operation state, it is necessary to consider the effects and interaction of other phenomena on the structure. In this case the hydraulic effect of water seepage into the dam material, this induces pore pressures in addition to the mechanical induced deformation thus generating additional stresses which may impair the safe operation of the dam. Fluid flow through a porous medium is assumed to be incompressible with constant material densities along the motion of the fluid through varying pressure points in the domain. Material permeability is assumed to be isotropic, with displacement, u describing the main variable for the mechanical process and pore water pressure, p_w describing the main variable for the hydraulic process.

The mathematical modeling of the fluid flow structure interaction involves the momentum conservation law and fulfilling the mass balance of liquid law (Lahmer, 2010). This is best understood from the effective stress law (Eq. (5)) which describes the contribution of the hydraulic effects of the pore pressures on the normal components of the mechanical stresses. When considering that negative stresses describes the compression state and positive stress the tension state, the effective stress is written as in Eq. (5) .

$$\sigma' = \sigma + \alpha m p_w \tag{5}$$

where:

- σ' is the effective stress tensor $[N/m^2]$
- p_w is the water pore pressure vector $[N/m^2]$
- α is the Biot's constant (usually $\alpha = 1$)
- m is specified unit tensor ($m = [1, 1, 1, 0, 0, 0]^T$)

The total discrete equation for the fluid-structure medium can be expressed in 10.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + \int_\Omega [B]^T \sigma d\Omega = \{f_{(t)}\} \tag{6}$$

$$\int_\Omega [B]^T \sigma d\Omega = \int_\Omega [B]^T \sigma' d\Omega + \int_\Omega [B]^T \alpha m p_w d\Omega \tag{7}$$

$$[K]\{u\} \approx \int_\Omega [B]^T \sigma' d\Omega \tag{8}$$

$$[Q]\{p\} \approx \int_\Omega [B]^T \alpha m p_w d\Omega \tag{9}$$

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} - [Q]\{p\} = \{f_{(t)}\} \tag{10}$$

Whereas from the mass fluid balance the seepage equation of fluid through the solid structure is given in Eq. (11).

$$[Q]^T\{\dot{u}\} + [S]\{\dot{p}\} + [H]\{p\} = \{q\} \tag{11}$$

The linear version of the equations in Eq. (10) and Eq. (11) for the dynamic coupled hydro-mechanical problem is given in Eq. (12).

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ Q^T & S \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f \\ q \end{Bmatrix} \tag{12}$$

The coupling, permeability and compressibility matrices Q, S and H are defined in Eq. (13 - 15) whereas the mass, damping and stiffness matrices M, C and K have been defined in Eq. (1).

$$Q_e = \int_{\Omega_e} B^T \alpha m N_p \, d\Omega \tag{13}$$

$$H_e = \int_{\Omega_e} (\nabla N_p)^T \, \kappa \, \nabla N_p \, d\Omega \tag{14}$$

$$S_e = \int_{\Omega_e} N_p^T \left(\frac{\alpha - n}{K_s} + \frac{n}{K_w} \right) N_p \, d\Omega \tag{15}$$

3. Forward Modeling

In order to identify the material properties, it is necessary to have a running forward model that simulates the observed phenomena. In this case the formulations in section 2 and Eq. (12) are implemented for a hypothetical dam using the material properties in Table I and II.

Table I. Mechanical dam properties.

$Height$ $H\,[m]$	$Breadth$ $B\,[m]$	$Length$ $L\,[m]$	$Youngs'\,modulus$ $E\,[MPa]$	$Poissons'ratio$ ν	$Density$ $\rho\,[kg/m^3]$	$Rayleigh$ α	$coeff.$ β	$Newmark$ β^*	$-\beta$ γ^*
28	19	0.1	2.4×10^3	0.15	24×10^2	-0.354	0.0162	0.5	0.25

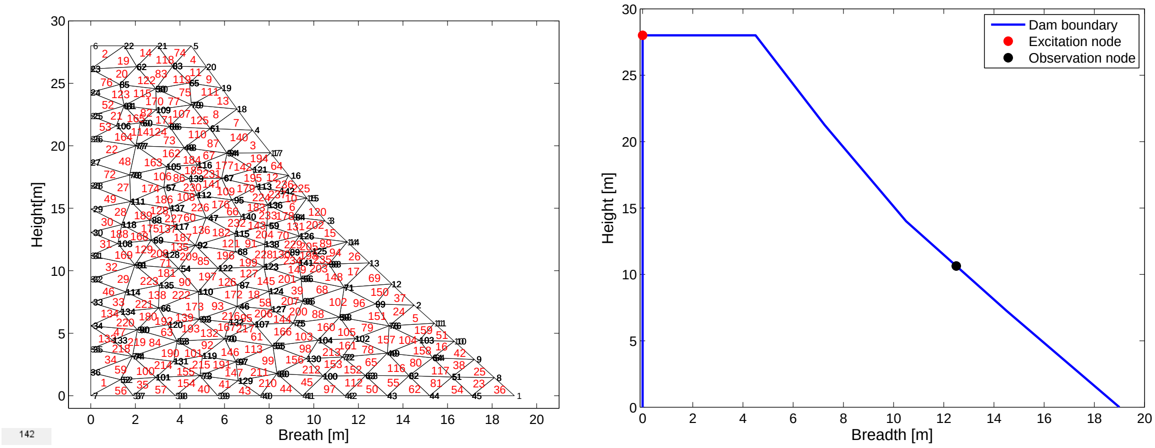
For this analysis, a dynamic force is induced on the upstream part of the dam crest and the response is recorded at an observation point as shown in Figure 1. The basic properties of the dynamic force can also be seen in Figure 2. The total excitation time is 4 $[s]$ during which peak amplitudes are reached and then the excitation is stopped, a time-step of 0.1 $[s]$ is used for the

Table II. Hydraulic dam properties.

$Water\ density$ $\rho_w\,[kg/m^3]$	$Fluid\,bulk\,modulus$ $K_f\,[N/m^2]$	$Porosity$ n	$Viscosity$ $\mu\,[N/s^2]$	$Permeability$ $\kappa\,[m^2]$	$Fluid\,flux$ $q\,[m^2/s]$
1000	2.2×10^6	1×10^{-6}	1×10^{-3}	1×10^{-18}	1.12×10^{-12}

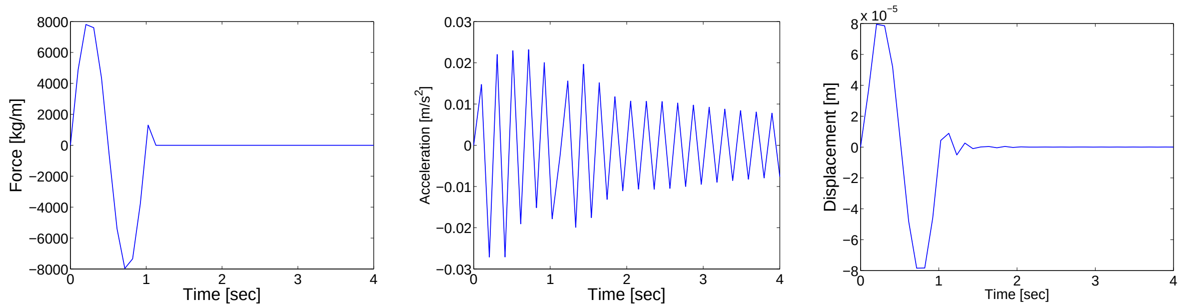
numerical integration of the mechanical part. For the solution of the hydraulic part of the coupled equation, a time-step of 1 $[s]$ is used.
Ideally, Eq. (11) is first solved and the resulting pore pressures are updated in Eq. (10), the resultant displacements due to the coupled hydro-mechanical action are then computed. However, in this case the effect of mechanical velocity ($\dot{u}$) on Eq. (11) is not considered.

To further investigate the effect of the hydro-mechanical coupling on the dynamic response of the dam, a comparison is made in Figure 3 between the frequency response of the mechanical system and the hydro-mechanical system for both the undamped and damped cases.



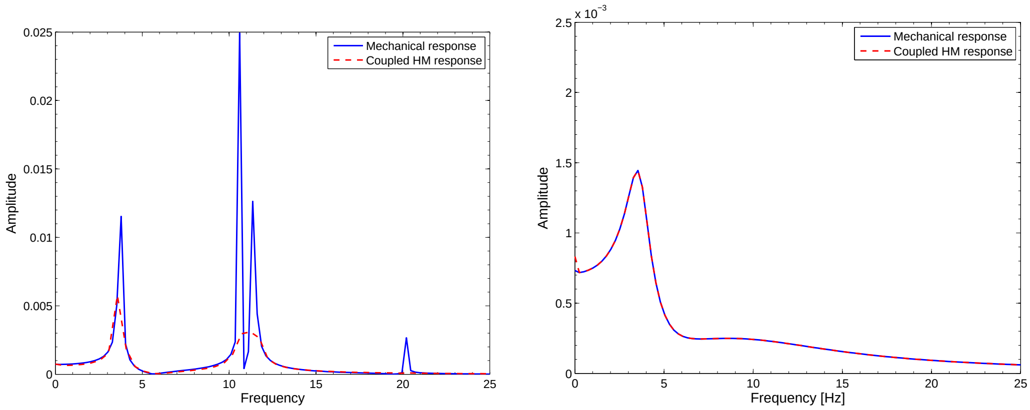
(a) Dam geometry and mesh　　(b) Dam excitation and observation points

Figure 1. Dam geometry.



(a) Excitation force　　(b) Acceleration　　(c) Excitation displacement

Figure 2. Dynamic load on dam.



(a) Frequency response for undamped case (b) Frequency response for damped case

Figure 3. Comparison between coupled and uncoupled frequency response at excitation node.

4. Inverse Modeling

The objective of the inverse analysis is to obtain reliable parameter values by minimizing the error between experimental data (describing in most cases a phenomena) and numerical results obtained from a mathematical model developed to simulate (as close as possible) the natural phenomena. Strategies based on different concepts exist for the calibration and optimization of models in engineering. These includes the Newton methods, which are part of the gradient methods with line search strategies used in unconstrained nonlinear optimization that gives very fast results when the initial guess is close to the minimum. The Nelder–Mead method is another of the direct search or gradient free strategies for locating the minima of a function. The algorithm was originally published in 1965 and according to (Singer and Nelder, 2009), it is one of the best known algorithms for multidimensional unconstrained optimization without derivatives. The Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by (Eberhart and Kennedy, 1995), inspired by social behavior of bird flocking or fish schooling. It is a heuristic based global optimizer easily applicable to a number of situations.

In this section an attempt is made on identifying the material parameters: Youngs' modulus E and Permeability κ of the dam from FE model discussed in previous sections. The material parameters are identified by minimizing the error between the experimental data and model results using the Nelder-mead algorithm which can be easily implemented in matlab.
The objective function being minimized (Eq. (16))is the sum of squared error with respect horizontal acceleration calculated at node 13 which located at about $1/3^{rd}$ of the dams' height from the base.

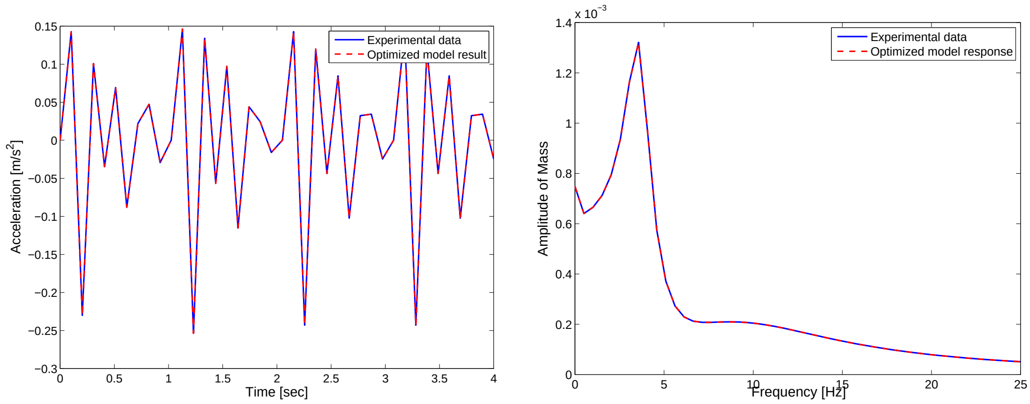
$$C_f(p, nn) = \sum_{i}^{n} (\ddot{U}^{\,mod}_{(t_i,p,nn)} - \ddot{U}^{\,exp}_{(t_i)})^2 \qquad (16)$$

where:

- C_f is the cost function to be minimized

- $\ddot{U}^{mod}_{(t_i,p,nn)}$ is the horizontal acceleration calculated at node (in this case 13) by the model
- $\ddot{U}^{exp}_{(t_i)}$ is the horizontal acceleration obtained from the experiment
- p is a vector of M, K and C
- t_i is the time step at each point
- n is the total number of data points

After several iterations the values of E and κ were obtained. Although the value of κ is unrealistic, a good fit is observed between the experimental response and simulated response. Figure 4 shows the quality of fit obtained after parameter optimization. The calculated values were $E = 2.4 \times 10^9$ and $\kappa = -2.64 \times 10^-17$, with the value of E being same as in Table I.



(a) Nodal acceleration (b) Frequency response

Figure 4. Comparison between experimental and model response for HM coupling.

The solutions obtained in this section would serve as "synthetic experimental data" for the inverse analysis to identify the dam material properties. In practice the acquisition of such data is done via acceleration sensors (Geo-phones) or other types of sensors instrumented on the dam to record various types of vibrations. This data after undergoing processing is used for the inverse modeling.

5. Conclusion

The damping effect of water on the dam response considering the hydro-mechanical coupling can be observed in the frequency response plots for the undamped case in Figure 3(a), this effect is minimal when structural damping is considered as obtain in Figure 3(b). To reduce computation cost the acceleration data was used as objective function instead of the frequency response which required more time. However in situations where data from acceleration sensors are available, these data are usually voluminous and may be difficult to fit, thus a frequency based objective function would be required to give better quality results after minimization.

Furthermore during the optimization, it was observed that no unique value of κ could be obtained although $E = 2.4 \times 10^9$ was obtained. This shows that κ is less sensitive as a result of the very low permeability of concrete. The ill-posed nature of the model is also observed here which would have been more pronounced if the dam material was more porous. In such cases, a regularization parameter Γ introduced to the objective function according to (Tikhonov, 1963) would improve the stability of the results as was obtained in (Alalade et al., 2015).

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