Reliable Power Flow Analysis of Systems with Uncertain Data

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Abstract: The load flow is the most basic tool for investigating the requirements of a power system and understanding its performance with fixed transmission line parameters and for specified sets of load and generation values. The common formulation of the power flow problem has all the input data specified at a specific time and conditions with fixed values. However, these data are approximate and do not take the measurement errors in transmission line and variation in load demand into consideration. When the input conditions are uncertain, a reliable load flow solution is needed that incorporates the effect of data uncertainty. This paper addresses the problem of uncertainties in the input parameters by specifying them as compact intervals, taking into consideration the errors in modeling and measurement of transmission line parameters and also the continuous influence of load measurement errors and fluctuation in the load demand. The load flow or power flow equations are modeled as a set of nonlinear algebraic equations. These systems of equations are first linearized using Taylor Series expansion and the solution is obtained by the Krawczyk's method of interval arithmetic. The proposed methodology is implemented in MATLAB environment using the INTLAB toolbox. The method is applied to 3 bus, 14 bus, 30 bus and 57 bus IEEE test systems. The results obtained are bounded and thus help in providing an insight to the operation and future expansion of the power system. These results are also compared with those obtained with another iterative method for solving interval linear equation systems.

Keywords: interval mathematics, load flow analysis, uncertain data, Newton Raphson method, Intlab toolbox, Krawczyk's method

1. Introduction

The optimal load flow problem has had a long history in its development for more than 25 years. A generalized formulation of the economic dispatch problem including voltage and other operating constraints was introduced and was later named the optimal power or load flow problem (OPF) (Abdel-Hady and Abdel-Aal Hassan, 2012). Power flow studies are the backbone of power system analysis and design. They are necessary in planning and designing the future expansion of power systems as well as in determining the best operation of existing systems.

Modern power systems are operating under highly stressed and unpredictable conditions because of many issues like market-oriented reforms, consumer utility, measurement errors, use of renewable generation and electric vehicles, etc. The changes resulting from these uncertain factors lead to higher requirements for the reliability of power grids. In this situation, conventional methodologies cannot be applied, so robust and reliable methods become very essential.

The load flow problem in an electric power system is concerned with solving a set of static nonlinear equations describing the electric network performance. The problem is formulated on the basis of Kirchhoff's laws in terms of active and reactive power injections and voltages at each node in the system. Load flow studies or power flow studies is the basic tool for investigating the requirements of power system, viz., generation should be sufficient to meet demand and losses, bus voltages should be in specified range and reactive powers limits of generator buses should be within limits (Grainger and Stevenson, 1994). This information is essential for planning for future expansion such as adding new generator sites to meet increased load demand, operation of the current state of the system, exchange of power between utilities, etc.

The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flow in each line (Hadi Saadat, 2002). As four quantities are involved, four independent constraints are required. In the load flow problem, the buses in the power grid are generally divided into three categories, i.e., slack bus, load bus PQ, and regulated or generator bus PV. A slack bus which is taken as reference is specified with both bus voltage magnitude and phase angle. A PQ bus which can typically be a substation or a power plant with fixed real and reactive power is specified with both active and reactive power injections. A PV bus which is usually a substation with adjustable reactive compensation devices or a power plant with reactive reserves is specified with real power injection and bus voltage magnitude. In reality, a power plant that has adequate capacity and is responsible for frequency control is often selected as a slack bus (Almeida et. al, 1994).

The mathematical formulation of the load flow problem results in a system of algebraic nonlinear equations where the input data are considered to be fixed for all time at all system conditions. However, in reality the models used in power flow analysis are only approximations. The network parameters are usually approximated even though the uncertainties may arise from variances in the model parameters of transmission system elements, such as resistance, reactance and/or capacitance values due to environmental conditions and measurement errors. Also, the specified variables, like real power at PV buses, are inaccurate as they may have measurement errors (Wang and Alvarado, 1994). Further, the demands can vary in a fast and disordered way. Moreover, the uncertainty in the input data can be enlarged due to both rounding and truncating processes that occur in numerical computations (Barboza, Dimuro and Reiser, 2004). Therefore, the final results obtained by conventional methods are wrongly implemented in the system. So, a more reliable power flow algorithm is required which allows the analysts to incorporate both the estimate in data and solution tolerance, i.e., the uncertainty in input parameters and the effect of propagation of data inaccuracies, thus obtaining a range of values for each output quantity (Vaccaro et.al, 2009; Vaccaro et. al, 2013).

Researchers and power engineers have recognized the importance of these uncertainties. Wang and Alvarado are the pioneers in this field where the authors have solved the interval nonlinear equations using the Newton operator and the Gauss Seidel method in (Wang and Alvarado, 1994). Later, in (Barboza, Dimuro and Reiser, 2004) load uncertainty has been dealt by solving the nonlinear equations using Krawczyk's method (Moore, 1966). Recently, many authors have used optimization techniques to address the existence of uncertainty in power flow problem (Dimitrovski and Tomsovic, 2004; Vaccaro et.al, 2009; Vaccaro et. al, 2010; Vaccaro et. al, 2013).

In order to overcome the aforesaid limitations and also control these numerical errors, we propose to apply the technique of interval arithmetic in the Newton-Raphson approach of power flow analysis. In this paper, we develop a methodology by taking into account the various uncertainties. Here, the input parameters are represented as intervals taking into consideration

- the errors in modeling and measurement of transmission line parameters.
- influence of the load measurement errors and fluctuation in the demand.

The nonlinear system is first linearized by the Newton Raphson method and then Krawczyk's method of interval mathematics is applied to solve the system of linearized equations. The implementation is performed in MATLAB environment, using the INTLAB toolbox developed by S. Rump (Rump, 1999). The proposed methodology is tested on 3 bus, 14 bus, 30 bus and 57 bus IEEE test systems.

The rest of the paper is organized as follows: Section II reviews some basic concepts related to interval arithmetic and interval systems. In Section III, we give the main characteristics of load flow problem. In Section IV we describe the interval arithmetic applied to the load flow problem with the proposed approach. In Section V, we give the results on the test problems followed by conclusion section.

2. Interval Arithmetic

Interval arithmetic, interval mathematics, interval analysis, or interval computation, is a method developed by mathematicians since the 1950s and 1960s as an approach to putting bounds on rounding errors and measurement errors in mathematical computation, thus developing numerical methods that yield reliable results. It is an arithmetic developed by R. E. Moore that is defined on sets of intervals instead on sets of real numbers (Moore, 1966). It combines interval arithmetic with analytic estimation techniques to compute the sharpest possible interval solution set which completely contains the true solution set. The power of interval arithmetic lies in its implementation on computers. It solves problems which are unsolvable by non-interval methods and has been used recently for global optimization, solving ordinary differential equations, linear systems, optimization, etc. Interval arithmetic is a logical extension of standard arithmetic that uses operators defined over real intervals.

Following the notations given by (Moore, 1966) and (Ralhan and Ray, 2013), let,

$$\mathbf{x} = [a, b] | a \leq b, a, b \in R \tag{1}$$

be a real interval where a is the infimum (lower endpoint) and b is the supremum (upper endpoint) of $\mathbf{x}$. The width of interval is defined as $w(\mathbf{x})= a - b$. The midpoint of the interval is defined as $m(\mathbf{x})= (a + b)/2$. For a n dimensional interval vector $\mathbf{x}^* = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n]$, the midpoint of interval vector $\mathbf{x}^*$ is given by $m(\mathbf{x}^*)=[m(\mathbf{x}_1), m(\mathbf{x}_2), ..., m(\mathbf{x}_n)]$. The width of interval vector is $w(\mathbf{x}^*)=[w(\mathbf{x}_1), w(\mathbf{x}_2), ..., w(\mathbf{x}_n)]$. A degenerate interval has both its lower and upper endpoints same.

Let $\mathbf{x}= [a, b]$ and $\mathbf{y}= [c, d]$ be two intervals. Let $+$, $-$, $*$ and $/$ denote the operation of addition, subtraction, multiplication and division, respectively. If $\otimes$ denotes any of these operations for the arithmetic of real numbers x and y, then the corresponding operation for arithmetic of interval numbers $\mathbf{x}$ and $\mathbf{y}$ is,

$$\mathbf{x} \otimes \mathbf{y}=[\mathbf{x} \otimes \mathbf{y} \mid x \in \mathbf{x}, y \in \mathbf{y}]$$

The above definition is equivalent to the following rules:

$$\begin{aligned}
\mathbf{x}+\mathbf{y} &= [a+c, b+d] \\
\mathbf{x}-\mathbf{y} &= [a-d, b-c] \\
\mathbf{x}*\mathbf{y} &= [\min(ac, bc, ad, bd), \max(ac, bc, ad, bd)] \\
\mathbf{x}/\mathbf{y} &= [\mathbf{x}*[1/d, 1/c]], 0 \notin \mathbf{y}
\end{aligned}$$

.

An interval function $F(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$ of intervals $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ is an interval valued function of one or more variables. $F(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$ is said to be an interval extension of a real function $f(x_1, x_2, ..., x_n)$ if $f(x_1, x_2, ..., x_n) \in \mathbf{F}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$, whenever $x_i \subset \mathbf{x}_i$ for all $i = 1, 2, ..., n$. F is said to be inclusion monotonic if,

$$x_i \subset y_i \Longrightarrow F(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n) \subset F(\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_n) \tag{2}$$

.

Also, $F(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$ contains the range of $f(x_1, x_2, ..., x_n)$. Interval functions $F(\mathbf{x})$ can be constructed in any programming language in which interval arithmetic is implemented, viz., C/C++ and Fortran 90/95, Maple, MATLAB, etc. However, the computations are slow and costly. Computing an interval bound carries a cost of 2 to 4 times as much effort as evaluating $f(\mathbf{x})$ (Moore, 1966; Hansen, 1992). INTLAB implemented with MATLAB enables basic interval operations to be performed on real and complex interval scalars, vectors and matrices.

3. Load Flow Review

The power flow equations (Hadi Saadat, 2002; Chen et. al, 2008) to describe an n bus system shown in Figure 1, are formulated using the bus admittance matrix Y as:

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \tag{3}$$

where I_i is the current entering in bus i and Y_{ij} is the admittance between the i^{th} and j^{th} buses. These equations are established using the bus analysis which results in node voltages as independent variables.

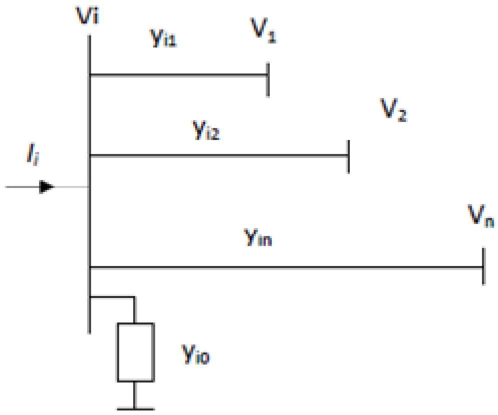

Figure 1. A Typical bus of Power system.

In polar form Eq.(3) is expressed as,

$$I_i = \sum_{j=1}^{n} |Y_{ij}|\angle\theta_{ij}|V_j|\delta_j \tag{4}$$

where, $|Y_{ij}|$ is the magnitude and $\angle\theta_{ij}$ is the angle of the admittance $|Y_{ij}|$ and $|V_j|$ is the magnitude and $\angle\delta_j$ is the phase angle of the voltage. The complex power at bus i is,

$$P_i - jQ_i = V_i^* I_i \tag{5}$$

P_i is the net real power injection and Q_i is the net reactive power injection at bus i

Again, from Eqs.(4) and (5),

$$P_i - jQ_i = |V_i|\angle - \delta_i \sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j \tag{6}$$

Thus,

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{7}$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{8}$$

The general practice in power flow studies is to identify the three types of buses in the network. At each bus i, out of the four quantities δ_i, $|V_i|$, P_i, and Q_i, two are specified and the remaining two are determined. Specified quantities are chosen according to the criteria explained in the following subsections.

3.1. Load Buses

At each non generator bus i, called a load bus or a PQ bus, both P_{gi} and Q_{gi} are zero and P_{di} and Q_{di} drawn from the system by the load are known from historical record, load forecast, or measurement, where the suffixes g and d denote generator and demand. Therefore, the two unknown quantities those to be determined are δ_i and $|V_i|$. Initially δ_i and $|V_i|$ are set as 0.0 and 1.0, respectively.

3.2. Voltage Controlled Buses

Any bus of the system at which the voltage magnitude is kept at a constant is said to be a voltage-controlled bus. A generator bus is usually called a voltage-controlled or a PV bus. A prime mover of any generator can control the amount of generated megawatts (MW), whereas generator voltage magnitude can be controlled by generators excitation system. Therefore, at each generator bus i both $|V_i|$ and P_{gi} may be properly specified. The two unknown quantities that must be determined are δ_i and Q_i. Initially δ_i is taken as 0.0.

3.3. Slack Bus

The voltage angle of a slack bus serves as a reference for the angles of all other bus voltages. Thus, the usual practice is to set δ_i to zero degree. Voltage magnitude of the slack bus $|V_i|$ is also specified. Therefore, the two unknown quantities P_i and Q_i must be determined during the load flow analysis.

3.4. Conventional Load Flow Problem Formulation

The set of equations given by Eq.(7) constitute a set of nonlinear algebraic equations as a function of voltage magnitude in per unit and phase angle in radians. Generalizing,

$$\begin{aligned} \mathbf{P}(|\mathbf{V}|, \delta) &= \mathbf{P}_s \\ \mathbf{Q}(|\mathbf{V}|, \delta) &= \mathbf{Q}_s \end{aligned} \tag{9}$$

If $(\mathbf{V}_k, \delta_k)$ is an initial estimate and $\Delta\mathbf{V}$ and $\Delta\delta$ are small deviations in voltage magnitudes and angles respectively, except the slack bus then,

$$\begin{aligned} \mathbf{P}(\mathbf{V}_k + \Delta\mathbf{V}, \delta_k + \Delta\delta) &= \mathbf{P}_s \\ \mathbf{Q}(\mathbf{V}_k + \Delta\mathbf{V}, \delta_k + \Delta\delta) &= \mathbf{Q}_s \end{aligned} \tag{10}$$

Expanding Eq.(10) in Taylor series about an initial estimate $(\mathbf{V}_k, \delta_k)$ and neglecting higher order terms, we obtain,

$$\begin{aligned} \mathbf{P}_{\mathbf{V}_k,\delta_k} + \Delta\mathbf{V}\frac{\partial \mathbf{P}_{\mathbf{V}_k,\delta_k}}{\partial \mathbf{V}} + \Delta\delta\frac{\partial \mathbf{P}_{\mathbf{V}_k,\delta_k}}{\partial \delta} - \mathbf{P}_s = 0 \\ \mathbf{Q}_{\mathbf{V}_k,\delta_k} + \Delta V\frac{\partial \mathbf{Q}_{\mathbf{V}_k,\delta_k}}{\partial \mathbf{V}} + \Delta\delta\frac{\partial \mathbf{Q}_{\mathbf{V}_k,\delta_k}}{\partial \delta} - \mathbf{Q}_s = 0 \end{aligned} \tag{11}$$

Hence,

$$\begin{bmatrix} \mathbf{P}_s - \mathbf{P}_{\mathbf{V}_k,\delta_k} \\ \mathbf{Q}_s - \mathbf{Q}_{\mathbf{V}_k,\delta_k} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{P}_{\mathbf{V}_k,\delta_k}}{\partial \delta} & \frac{\partial \mathbf{P}_{\mathbf{V}_k,\delta_k}}{\partial \mathbf{V}} \\ \frac{\partial \mathbf{Q}_{\mathbf{V}_k,\delta_k}}{\partial \delta} & \frac{\partial \mathbf{Q}_{\mathbf{V}_k,\delta_k}}{\partial \mathbf{V}} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|\mathbf{V}| \end{bmatrix} \tag{12}$$

Elements of the Jacobian matrix are the partial derivatives of Eqs.(7) and (8) for $i = 2, ..., n$, evaluated at $(\mathbf{V}_k, \delta_k)$ and bus 1 is assumed to be the slack bus. Using the initial estimates given in Sections 3.1, 3.2 and 3.3, $\mathbf{P}_{\mathbf{V}_k,\delta_k}$ and $\mathbf{Q}_{\mathbf{V}_k,\delta_k}$ are calculated using Eqs.(7) and (8) respectively for each bus. Writing in compact form,

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|\mathbf{V}| \end{bmatrix} \tag{13}$$

Therefore,

$$\begin{bmatrix} \Delta\delta \\ \Delta|\mathbf{V}| \end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} \tag{14}$$

Expressing Eq.(14) as,

$$\mathbf{Jx} = \mathbf{b} \tag{15}$$

where,

$$\mathbf{x} = \begin{bmatrix} \Delta\delta \\ \Delta|\mathbf{V}| \end{bmatrix} \tag{16}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \tag{17}$$

and

$$\mathbf{b} = \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} \tag{18}$$

The above equations have inherent assumptions which include the factors, such as the three phase system is in balanced, steady-state condition so that the frequency and voltage are constant, the real and reactive power demands are known and specified precisely, the real power injection and voltage magnitude of generators are fixed, and the network topology and impedances are known precisely. But these are not valid, at least in true sense. The uncertainties may be induced by modeling errors due to the approximations in the values of the resistances, reactances and shunts in the models which are used to represent transmission lines and transformers. The practical power systems are very large with tens of thousands of buses. As a consequence, the probability of data errors increases dramatically with system size. Therefore, we take the data uncertainty into consideration by characterizing the load and transmission line parameters as a range of real values or a real interval instead of crisp values.

4. Load Flow Solution using Interval Method

As the uncertainties in the data could affect the deterministic power flow solution to a considerable extent, reliable solution algorithms that incorporate the effect of data uncertainties into the load flow analysis are therefore required. In this way, the uncertainty propagation effect is explicitly considered and the bounded values for power flow studies can be assessed. Thus, the voltages, angles and powers are obtained as intervals that include all computational errors and all the possible results obtained by computations with real numbers. The known real and reactive power injections P_i, Q_i, as well as other input line parameters are modeled as intervals that are estimated in the beginning of the process. The magnitude and angle of voltages are unknown intervals to be determined for all buses except the slack bus. In order to reach the solution, we use the Krawczyk method which is an iterative method to solve interval linear system of equations (Moore, Baker and Cloud, 2008; Hansen, 1992).

4.1. Krawczyk Method

The power flow equations (Eqs.(13), (14), and (15)) have been obtained after linearization using Taylor series expansion method. Since we have taken the uncertainty in the transmission line parameters, i.e, resistances and reactances, the Jacobian **J** is an interval matrix. Also, the variations in power lead to interval vectors (which are actually nonlinear in nature but are linearized). By using the Krawczyk method, along with interval arithmetic we can obtain the solution of the finite system of linear equations. Consider a system of finite system of linear equations represented as,

$$\mathbf{Jx} = \mathbf{b} \tag{19}$$

$$i,e., \quad \mathbf{x} = \mathbf{J}^{-1}\mathbf{b} \tag{20}$$

Let $\mathbf{Y}$ be an approximate inverse of $\mathbf{J}$, i.e.,

$$\mathbf{Y} \simeq \mathbf{J}^{-1} \tag{21}$$

Multiplying both sides of Eq.(19) by $\mathbf{Y}$. We have,

$$\mathbf{YJx} = \mathbf{Yb} \tag{22}$$

From Eq.(20) and Eq.(22), we get,

$$[\mathbf{I} - \mathbf{YJ}]\mathbf{x} = \mathbf{J}^{-1}\mathbf{b} - \mathbf{Yb} \tag{23}$$

Let us consider,

$$\mathbf{E} = \mathbf{I} - \mathbf{YJ} \tag{24}$$

Therefore, Eq.(23) can be written as,

$$\mathbf{Ex} = \mathbf{x} - \mathbf{Yb} \tag{25}$$

or,

$$\mathbf{x} = \mathbf{Yb} + \mathbf{Ex} \tag{26}$$

The norm of an interval matrix say $\mathbf{A}$ is given by,

$$||\mathbf{A}|| = max_i \sum_j |\mathbf{A}_{ij}| \tag{27}$$

Since $\mathbf{E}$ is an interval matrix $||\mathbf{E}|| = \max_i \sum_j |E_{ij}|$. So if $||\mathbf{E}|| \leq 1$ using Eq.(27), then the sequence,

$$\begin{aligned} \mathbf{x}^{(k+1)} &= \mathbf{Yb} + \mathbf{Ex}^{(k)} \cap \mathbf{x}^{(k)}; k = 0, 1, 2, ... \\ \mathbf{x}_i^{(0)} &= [-1, 1]||\mathbf{Yb}||/(1 - ||\mathbf{E}||); i = 0, 1, ..., n \end{aligned} \tag{28}$$

is a nested sequence of interval vectors containing the unique solution to Eq.(19) for every interval matrix $\mathbf{J}$ and every interval vector $\mathbf{b}$. From Eq.(28) the new estimates for the bus voltage angle and magnitude are respectively obtained as,

$$\delta^{(k+1)} = \delta^(k) + \Delta\delta^(k) \tag{29}$$

$$|V^{(k+1)}| = |V^{(k)}| + \Delta|V^{(k)}| \tag{30}$$

Since nonlinearities in the power system can be said to be encompassed by intervals in the linearized system (Kearfott, 1991; Hansen, 1992), Eqs.(29) and (30) give us a bounded and converging solution for the non linear power flow equations.

5. Results

In this section we test the performance of the proposed method. For this we have carried out the tests on these standard systems, namely IEEE 3 bus, 14 bus, 30 bus and 57 bus test power systems from the Archive, by conventional load flow and interval load flow methods. We specify the loads and generation in Mega Watt (MW) and Mega Volt Ampere Reactive (MVAR), respectively, bus voltages in per unit, and their angles in degrees. Loads and generations are converted into per unit quantities on the base Mega Volt Ampere (MVA) selected. Three types of tests are performed on all the systems

1. Available data has been taken as fixed data without any uncertainty
2. Measurement error of 2% in the transmission line parameters was considered.
3. A 10% variation in active and reactive powers of load and generator were carried out.

The test cases considered are as given in the subsequent subsections. In this paper, we implement the methods in MATLAB environment using the INTLAB toolbox (Rump, 1999).

5.1. IEEE 3 bus system

The 3 bus network shown in Figure 2 consists of one load bus (PQ bus, bus 2), one generator bus (PV bus, bus 3) and a slack bus (reference bus, bus 1) and has three circuits whose parameters,

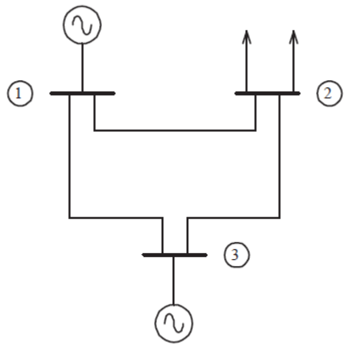

Figure 2. 3 Bus Test Case.

Table I. Line data for 3 bus test case.

Bus	Bus	$R_{p.u.}$	$X_{p.u.}$	$C(MVar)$	$Tr.TapSet.$
1	2	0.02	0.04	0	1
1	3	0.01	0.03	0	1
2	3	0.0125	0.025	0	1

i.e., resistance, reactance and capacitance are shown in Table I. Table II shows power flow data for the test case. Initially, we consider that the power at load bus is accurate and measurement errors are equal to zero.

With conventional Newton Raphson method we obtain the results as shown in Table III without considering any uncertainty in the input data. In the table we show the magnitude and angle of voltages at all system buses ($|V|$ is magnitude of bus voltage, δ is the angle in degrees), P_D and Q_D, P_G and Q_G are the active and reactive powers of load, generator and slack buses.

Then, we consider the existence of uncertainties in the values of the input data as mentioned above and Table IV and Table V show the results of the simulations performed using interval arithmetic. The active power at the slack bus for 2% variation in power load measurements is [1.4736, 2.8930] using intervals and by conventional method, it is 2.1482 p.u. which is inclusive in that range. Similarly, the reactive power at slack bus by conventional method is 1.4085 and interval value is [1.0008, 1.8217].

Table II. Bus data for 3 bus test case.

Bus	V(p.u.)	Angle (deg)	P_D	Q_D	P_G	Q_G	Q_{min}	Q_{max}	MVar
1	1.05	0	0	0	0	0	0	0	0
2	1.00	0	-4	-2.5	0	0	0	0	0
3	1.04	0	0	0	2	0	0	0	0

Table III. Load Flow Solution for 3 bus system with conventional Newton Raphson method.

Bus	$\|V\|$	δ(deg.)	P_D	Q_D	P_G	Q_G
1	1.05	0	0	0	2.1842	1.4085
2	0.97168	-0.04706	-4	-2.5	0	0
3	1.04	0.008705	0	0	2	1.4617

Table IV. Load Flow Solution for 3 bus Interval system with variation in load power.

Bus	$\|V\|$	δ(deg.)	Q_G
1	[1.0500, 1.0501]	[0.0000, 0.0000]	[1.0008, 1.8217]
2	[0.9637, 0.9797]	[-3.5040, -1.8889]	[0.0000, 0.0000]
3	[1.0400, 1.0401]	[-1.0924, 0.0948]	[0.1240, 2.7880]

5.2. IEEE 14 bus system

The 14 bus IEEE test case shown in Figure 3 has been analyzed in the same way. Table VI shows the results with the conventional Newton Raphson method. Table VII and Table VIII show the results of the simulations performed using Interval mathematics considering the existence of uncertainties in the values of the input data. The active power at the slack bus for 2% variation in power load measurements is [1.9931, 2.6583] using intervals and the interval value of reactive power at slack bus is [-0.2301, -0.0757].

5.3. IEEE 30 bus system

Next we consider 30 bus IEEE test case. Table IX and Table X show the results of the simulations performed using Interval mathematics considering the existence of uncertainties in the values of the input data. The active power at the slack bus for 2% variation in power load measurements

Table V. Load Flow Solution for 3 bus Interval system with variation in line parameters and load power.

Bus	$\|V\|$	δ(deg.)	Q_G
1	[1.0500, 1.0501]	[0.0000, 0.0000]	[1.0010, 1.8219]
2	[0.9637, 0.9797]	[-3.5024, -1.8880]	[0.0000, 0.0000]
3	[1.0400, 1.0401]	[-1.0918, 0.0949]	[0.1237, 2.7877]

Table VI. Load Flow Solution for 14 bus system with conventional Newton Raphson method.

Busno.	$V(p.u.)$	δ(deg.)	P_D	Q_D	P_G	Q_G
1	1.060	0.000	0.000	0.000	2.746	-0.238
2	1.045	-6.149	0.217	0.127	0.004	0.628
3	1.010	-13.781	0.942	0.190	0.000	0.272
4	1.014	-11.178	0.478	-0.039	0.000	0.000
5	1.017	-9.603	0.076	0.016	0.000	0.000
6	1.070	-15.283	0.112	0.075	0.000	0.216
7	1.050	-14.159	0.000	0.000	0.000	0.000
8	1.090	-14.159	0.000	0.000	0.000	0.246
9	1.034	-15.732	0.295	0.166	0.000	0.000
10	1.032	-15.935	0.090	0.058	0.000	0.000
11	1.047	-15.727	0.035	0.018	0.000	0.000
12	1.054	-16.136	0.061	0.016	0.000	0.000
13	1.047	-16.178	0.135	0.058	0.000	0.000
14	1.021	-16.952	0.149	0.050	0.000	0.000

Table VII. Load Flow Solution for 14 bus Interval system with variation in load power.

Bus	$\lvert V \rvert$	δ(deg.)	Q_G
1	[1.0600, 1.0601]	[0.0000, 0.0000]	[-0.2381, -0.0757]
2	[1.0449, 1.0450]	[-6.7794, -4.1951]	[-0.0671, 1.0016]
3	[1.0100, 1.0101]	[-14.3743, -11.1098]	[-0.1268, 0.6576]
4	[1.0110, 1.0176]	[-11.5647, -8.9499]	[0.0000, 0.0000]
5	[1.0143, 1.0202]	[-9.8895, -7.6405]	[0.0000, 0.0000]
6	[1.0700, 1.0701]	[-16.1462, -12.6894]	[-0.7583, 1.1850]
7	[1.0472, 1.0535]	[-14.8825, -11.6226]	[0.0000, 0.0000]
8	[1.0900, 1.0901]	[-14.8825, -11.6226]	[0.2152, 0.2748]
9	[1.0288, 1.0386]	[-16.6281, -13.0375]	[0.0000, 0.0000]
10	[1.0277, 1.0375]	[-16.8653, -13.2181]	[0.0000, 0.0000]
11	[1.0446, 1.0504]	[-16.6346, -13.0615]	[0.0000, 0.0000]
12	[1.0517, 1.0553]	[-17.1042, -13.4328]	[0.0000, 0.0000]
13	[1.0445, 1.0497]	[-17.1593, -13.4572]	[0.0000, 0.0000]
14	[1.0155, 1.0272]	[-18.0196, -14.1104]	[0.0000, 0.0000]

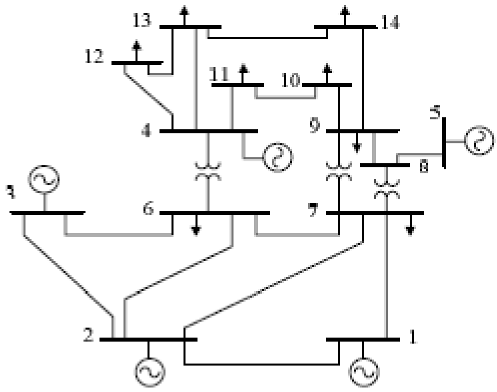

Figure 3. 14 Bus Test System.

Table VIII. Load Flow Solution for 14 bus Interval system with variation in line parameters and load power.

Bus	$\|V\|$	δ(deg.)	Q_G
1	[1.0600, 1.0601]	[0.0000, 0.0000]	[-0.2386, -0.0762]
2	[1.0449, 1.0450]	[-6.7826, -4.1989]	[-0.0669, 1.0019]
3	[1.0100, 1.0101]	[-14.3765, -11.1134]	[-0.1270, 0.6575]
4	[1.0110, 1.0176]	[-11.5672, -8.9534]	[0.0000, 0.0000]
5	[1.0143, 1.0202]	[-9.8922, -7.6440]	[0.0000, 0.0000]
6	[1.0700, 1.0701]	[-16.1464, -12.6910]	[-0.7584, 1.1849]
7	[1.0472, 1.0535]	[-14.8835, -11.6249]	[0.0000, 0.0000]
8	[1.0900, 1.0901]	[-14.8835, -11.6249]	[0.2152, 0.2749]
9	[1.0288, 1.0387]	[-16.6284, -13.0392]	[0.0000, 0.0000]
10	[1.0277, 1.0375]	[-16.8654, -13.2197]	[0.0000, 0.0000]
11	[1.0446, 1.0504]	[-16.6347, -13.0630]	[0.0000, 0.0000]
12	[1.0517, 1.0553]	[-17.1040, -13.4341]	[0.0000, 0.0000]
13	[1.0445, 1.0497]	[-17.1591, -13.4585]	[0.0000, 0.0000]
14	[1.0155, 1.0272]	[-18.0192, -14.1115]	[0.0000, 0.0000]

is [2.2425, 2.9780] using intervals which is inclusive in that range. Similarly, the reactive power at slack bus has interval value is [-0.2544,-0.0803] .

5.4. IEEE 57 bus system

Finally we consider 57 bus IEEE test case. In this case a measurement error of 2% in the transmission line parameters was assumed. Tables XI and XII show the results of the simulations performed using interval arithmetic considering the existence of uncertainties in the values of the input data. Next a 20% variation in active and reactive powers of load and generator were assumed. This was done

Table IX. Load Flow Solution for 30 bus Interval system with variation in load power.

Bus	$\lvert V \rvert$	δ(deg.)	Q_G
1	[1.0600, 1.0601]	[0.0000, 0.0000]	[-0.2544, -0.0803]
2	[1.0429, 1.0430]	[-6.3511, -4.6426]	[-0.0873, 1.0649]
3	[1.0186, 1.0244]	[-9.0298, -6.9783]	[0.0000,0.0000]
4	[1.0097, 1.0160]	[-10.9077, -8.4153]	[0.0000, 0.0000]
5	[1.0100, 1.0101]	[-16.1915, -12.5710]	[-0.2099, 0.9167]
6	[1.0096, 1.0145]	[-12.8548, -9.9412]	[0.0000, 0.0000]
7	[1.0011, 1.0058]	[-14.8170, -11.4820]	[0.0000,0.0000]
8	[1.0100, 1.0101]	[-13.6719, -10.5588]	[-0.6206, 1.2306]
9	[1.0464, 1.0556]	[-16.2159, -12.6518]	[0.0000, 0.0000]
10	[1.0365, 1.0522]	[-17.9750, -14.0733]	[0.0000, 0.0000]
11	[1.0820, 1.0821]	[-16.2159, -12.6518]	[0.1267, 0.1953]
12	[1.0530, 1.0617]	[-17.1766, -13.4283]	[0.0000, 0.0000]
13	[1.0709, 1.0710]	[-17.1766, -13.4283]	[0.0536, 0.1546]
14	[1.0362, 1.0487]	[-18.1762, -14.2064]	[0.0000, 0.0000]
15	[1.0307, 1.0449]	[-18.2662, -14.2903]	[0.0000, 0.0000]
16	[1.0381, 1.0513]	[-17.8166, -13.9444]	[0.0000, 0.0000]
17	[1.0311, 1.0472]	[-18.1645, -14.2121]	[0.0000, 0.0000]
18	[1.0193, 1.0366]	[-18.9491, -14.8180]	[0.0000, 0.0000]
19	[1.0160, 1.0345]	[-19.1381, -14.9658]	[0.0000, 0.0000]
20	[1.0203, 1.0383]	[-18.9045, -14.8000]	[0.0000, 0.0000]
21	[1.0229, 1.0413]	[-18.4906, -14.4464]	[0.0000, 0.0000]
22	[1.0235, 1.0418]	[-18.4724, -14.4368]	[0.0000, 0.0000]
23	[1.0183, 1.0361]	[-18.7160, -14.6088]	[0.0000, 0.0000]
24	[1.0112, 1.0319]	[-18.9230, -14.7373]	[0.0000, 0.0000]
25	[1.0098, 1.0279]	[-18.4728, -14.3749]	[0.0000, 0.0000]
26	[0.9902, 1.0122]	[-19.0010, -14.6834]	[0.0000, 0.0000]
27	[1.0185, 1.0330]	[-17.8839, -13.9411]	[0.0000, 0.0000]
28	[1.0080, 1.0135]	[-13.5871, -10.5278]	[0.0000, 0.0000]
29	[0.9965, 1.0154]	[-19.2754, -14.9973]	[0.0000, 0.0000]
30	[0.9837, 1.0053]	[-20.2658, -15.7635]	[0.0000, 0.0000]

to make a comparison of the performance of the proposed method with that of (Vaccaro et. al, 2013) and the results are shown in Tables XIII and XIV. From the results we can easily say that the bounds for the voltage and angles are comparable to that obtained in (Vaccaro et. al, 2013). The active power at the slack bus for 2% variation in power load measurements is [1.6903,5.9333] using the proposed technique which also includes the values obtained by the conventional method. Similarly, the reactive power at slack bus is [2.2970,3.1437].

Table X. Load Flow Solution for 30 bus Interval system with variation in variation in line parameters and load power.

Bus	$\|V\|$	δ(deg.)	Q_G
1	[1.0600, 1.0601]	[0.0000, 0.0000]	[-0.2554, -0.0813]
2	[1.0429, 1.0430]	[-6.3599, -4.6519]	[-0.0866, 1.0658]
3	[1.0186, 1.0244]	[-9.0411, -6.9902]	[0.0000,0.0000]
4	[1.0097, 1.0160]	[-10.9208, -8.4291]	[0.0000, 0.0000]
5	[1.0100, 1.0101]	[-16.1915, -12.5710]	[-0.2100, 0.9167]
6	[1.0096, 1.0145]	[-12.8703, -9.9576]	[0.0000, 0.0000]
7	[1.0011, 1.0058]	[-14.8317, -11.4978]	[0.0000,0.0000]
8	[1.0100, 1.0101]	[-13.6881, -10.5760]	[-0.6206, 1.2308]
9	[1.0464, 1.0556]	[-16.2297, -12.6667]	[0.0000, 0.0000]
10	[1.0365, 1.0522]	[-17.9878, -14.0874]	[0.0000, 0.0000]
11	[1.0820, 1.0821]	[-16.2297, -12.6667]	[0.1268, 0.1953]
12	[1.0530, 1.0617]	[-17.1885, -13.4416]	[0.0000, 0.0000]
13	[1.0709, 1.0710]	[-17.1885, -13.4416]	[0.0536, 0.1546]
14	[1.0362, 1.0487]	[-18.1879, -14.2195]	[0.0000, 0.0000]
15	[1.0307, 1.0449]	[-18.2780, -14.3036]	[0.0000, 0.0000]
16	[1.0381, 1.0513]	[-17.8287, -13.9580]	[0.0000, 0.0000]
17	[1.0311, 1.0472]	[-18.1770, -14.2260]	[0.0000, 0.0000]
18	[1.0193, 1.0366]	[-18.9609, -14.8313]	[0.0000, 0.0000]
19	[1.0160, 1.0345]	[-19.1500, -14.9792]	[0.0000, 0.0000]
20	[1.0203, 1.0383]	[-18.9166, -14.8137]	[0.0000, 0.0000]
21	[1.0229, 1.0413]	[-18.5032, -14.4604]	[0.0000, 0.0000]
22	[1.0236, 1.0418]	[-18.4851, -14.4508]	[0.0000, 0.0000]
23	[1.0184, 1.0361]	[-18.7280, -14.6224]	[0.0000, 0.0000]
24	[1.0112, 1.0319]	[-18.9357, -14.7514]	[0.0000, 0.0000]
25	[1.0099, 1.0279]	[-18.4867, -14.3903]	[0.0000, 0.0000]
26	[0.9903, 1.0122]	[-19.0147, -14.6987]	[0.0000, 0.0000]
27	[1.0185, 1.0330]	[-17.8987, -13.9573]	[0.0000, 0.0000]
28	[1.0080, 1.0135]	[-13.6048, -10.5465]	[0.0000, 0.0000]
29	[0.9965, 1.0155]	[-19.2896, -15.0131]	[0.0000, 0.0000]
30	[0.9837, 1.0053]	[-20.2796, -15.7789]	[0.0000, 0.0000]

6. Conclusion

In this study, a method for considering the uncertainties of the input parameters in the load flow solution for power systems has been presented. Based on interval arithmetic, the proposed methodology can consider the uncertainties in both the load demand and the transmission line parameters successfully. The solutions which we have obtained from the interval arithmetic based

Table XI. Load Flow Solution for 57 bus Interval system with variation in load power.

Bus no.	$V(p.u.)$	δ(deg.)	P_D	Q_D	P_G	Q_G
1	[1.06,1.06]	[0.00,0.00]	[0.55,0.55]	[0.17,0.17]	[1.69,5.93]	[2.29,3.14]
2	[1.00,1.00]	[-1.39,0.40]	[0.02,0.03]	[0.87,0.88]	[0.00,0.00]	[-2.16, -0.52]
3	[1.00,1.00]	[-8.89, -1.29]	[0.40,0.41]	[0.20,0.21]	[0.40,0.40]	[-4.00,4.72]
4	[0.99,0.99]	[-11.03, -1.53]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
5	[0.99,0.99]	[-13.58, -1.25]	[0.12,0.13]	[0.04,0.04]	[0.00,0.00]	[0.00,0.00]
6	[1.00,1.00]	[-14.31,-0.73]	[0.74,0.75]	[0.02,0.02]	[0.00,0.00]	[-4.85,5.55]
7	[0.98,0.99]	[-13.82,1.49]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
8	[1.00,1.00]	[-11.21,5.25]	[1.50,1.50]	[0.21,0.22]	[4.50,4.50]	[-4.10,3.23]
9	[1.00,1.00]	[-15.56, -0.99]	[1.20,1.21]	[0.25,0.26]	[0.00,0.00]	[-5.18,6.89]
10	[0.98,0.98]	[-16.43, -3.19]	[0.05,0.05]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
11	[0.98,0.98]	[-15.03, -2.27]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
12	[1.00,1.00]	[-14.95, -2.68]	[3.76,3.77]	[0.23,0.24]	[3.10,3.10]	[-4.47,5.05]
13	[0.97,0.98]	[-13.68, -2.59]	[0.17,0.18]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
14	[0.97,0.98]	[-12.43, -2.62]	[0.10,0.10]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
15	[0.99,1.00]	[-9.77, -2.12]	[0.21,0.22]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
16	[1.00,1.01]	[-12.11, -3.14]	[0.42,0.43]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
17	[1.02,1.03]	[-7.08, -2.30]	[0.41,0.42]	[0.08,0.08]	[0.00,0.00]	[0.00,0.00]
18	[0.98,0.99]	[-15.72, -5.26]	[0.27,0.27]	[0.09,0.09]	[0.00,0.00]	[0.00,0.00]
19	[0.95,0.97]	[-17.07, -6.00]	[0.03,0.03]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
20	[0.95,0.97]	[-17.11, -5.79]	[0.02,0.02]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
21	[0.991.01]	[-16.05, -4.71]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
22	[0.99,1.02]	[-15.93, -4.57]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
23	[0.99,1.01]	[-16.06, -4.59]	[0.06,0.06]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
24	[0.97,1.00]	[-17.31, -4.15]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
25	[0.92,0.96]	[-22.70, -8.59]	[0.06,0.06]	[0.03,0.03]	[0.00,0.00]	[0.00,0.00]
26	[0.94,0.96]	[-17.15, -3.80]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
27	[0.97,0.98]	[-17.00, -2.22]	[0.09,0.09]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
28	[0.98,1.00]	[-16.39, -1.20]	[0.04,0.04]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
29	[1.00,1.01]	[-15.93, -0.51]	[0.17,0.17]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
30	[0.90,0.95]	[-23.39, -9.16]	[0.03,0.03]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
31	[0.88,0.93]	[-24.22, -9.95]	[0.05,0.05]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
32	[0.91,0.95]	[-23.12, -9.56]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
33	[0.91,0.95]	[-23.17, -9.58]	[0.03,0.03]	[0.018,0.01]	[0.00,0.00]	[0.00,0.00]
34	[0.94,0.97]	[-17.64, -5.58]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
35	[0.95,0.98]	[-17.35, -5.43]	[0.05,0.06]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
36	[0.96,0.99]	[-17.03, -5.25]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
37	[0.97,1.00]	[-16.72, -5.09]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
38	[1.00,1.02]	[-15.67, -4.51]	[0.14,0.14]	[0.07,0.07]	[0.00,0.00]	[0.00,0.00]
39	[0.97,0.99]	[-16.79, -5.12]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
40	[0.96,0.99]	[-17.14, -5.30]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
41	[0.99,1.00]	[-18.99, -5.75]	[0.06,0.06]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
42	[0.95,0.97]	[-20.28, -6.90]	[0.07,0.07]	[0.04,0.04]	[0.00,0.00]	[0.00,0.00]
43	[1.01,1.02]	[-16.21, -3.31]	[0.02,0.02]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
44	[1.01,1.02]	[-14.75, -4.31]	[0.11,0.12]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
45	[1.03,1.04]	[-12.03, -3.21]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]

Table XII. Load Flow Solution for 57 bus Interval system with variation in load power (contd.).

Bus no.	$V(p.u.)$	δ(deg.)	P_D	Q_D	P_G	Q_G
46	[1.05,1.07]	[-13.67, -3.36]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
47	[1.02,1.04]	[-14.51, -3.63]	[0.00,0.00]	[0.29,0.29]	[0.00,0.00]	[0.00,0.00]
48	[1.01,1.03]	[-15.03, -4.01]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
49	[1.02,1.04]	[-16.32, -4.79]	[0.17,0.18]	[0.08,0.08]	[0.00,0.00]	[0.00,0.00]
50	[1.01,1.03]	[-17.47, -5.07]	[0.20,0.21]	[0.10,0.10]	[0.00,0.00]	[0.00,0.00]
51	[1.04,1.05]	[-17.42, -4.18]	[0.17,0.18]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
52	[0.96,0.98]	[-17.60, -1.80]	[0.04,0.04]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
53	[0.951,0.97]	[-18.29, -2.36]	[0.20,0.20]	[0.10,0.10]	[0.00,0.00]	[0.00,0.00]
54	[0.99,1.00]	[-17.83, -2.40]	[0.04,0.04]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
55	[1.04,1.05]	[-16.95, -2.05]	[0.06,0.06]	[0.03,0.03]	[0.00,0.00]	[0.00,0.00]
56	[0.94,0.97]	[-20.45, -7.28]	[0.07,0.07]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
57	[0.94,0.97]	[-20.91, -7.80]	[0.06,0.06]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]

Table XIII. Load Flow Solution for 57 bus Interval system with variation in load and generator power and in static reactive power.

Bus	$V(p.u.)$	δ(deg.)	P_D	Q_D	P_G	Q_G
1	[1.06,1.06]	[0.00,0.00]	[0.55,0.55]	[0.17,0.17]	[-0.42,8.06]	[2.00,3.72]
2	[1.00,1.00]	[-2.29,1.31]	[0.02,0.03	[0.87,0.88]	[0.00,0.00]	[-2.92,0.34]
3	[1.00,1.00]	[-12.69,2.50]	[0.40,0.41]	[0.20,0.21]	[0.40,0.40]	[-8.29,9.00]
4	[0.99,0.99]	[-15.78,3.20]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
5	[0.99,0.99]	[-19.74,4.90]	[0.12,0.13]	[0.04,0.04]	[0.00,0.00]	[0.00,0.00]
6	[1.00,1.00]	[-21.10,6.05]	[0.74,0.75]	[0.02,0.02]	[0.00,0.00]	[-10.44,10.89]
7	[0.98,0.99]	[-21.48,9.15]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
8	[1.00,1.00]	[-19.45,13.49]	[1.50,1.50]	[0.21,0.22]	[4.50,4.50]	[-10.65,8.50]
9	[1.00,1.00]	[-22.84,6.29]	[1.20,1.21]	[0.25,0.26]	[0.00,0.00]	[-12.92,14.76]
10	[0.97,0.99]	[-23.06,3.43]	[0.05,0.05]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
11	[0.97,0.99]	[-21.40,4.09]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
12	[1.00,1.000]	[-21.09,3.44]	[3.76,3.77]	[0.23,0.24]	[3.10,3.10]	[-9.96,9.89]
13	[0.97,0.99]	[-19.22,2.94]	[0.17,0.18]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
14	[0.96,0.98]	[-17.34,2.28]	[0.10,0.10]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
15	[0.99,1.00]	[-13.60,1.69]	[0.21,0.22]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
16	[1.00,1.01]	[-16.59,1.33]	[0.42,0.43]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
17	[1.01,1.03]	[-9.47,0.08]	[0.41,0.42]	[0.08,0.08]	[0.00,0.00]	[0.00,0.00]
18	[0.98,1.00]	[-20.95, -0.03]	[0.27,0.27]	[0.09,0.09]	[0.00,0.00]	[0.00,0.00
19	[0.94,0.98]	[-22.60, -0.47]	[0.03,0.03]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
20	[0.93,0.98]	[-22.77, -0.12]	[0.02,0.02]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]

Table XIV. Load Flow Solution for 57 bus Interval system with variation in load and generator power and in static reactive power (contd.).

Bus	$V(p.u.)$	δ(deg.)	P_D	Q_D	P_G	Q_G
21	[0.98,1.03]	[-21.72,0.95]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
22	[0.98,1.03]	[-21.61,1.10]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
23	[0.98,1.03]	[-21.79,1.14]	[0.06,0.06]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
24	[0.96,1.01]	[-23.89,2.42]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
25	[0.90,0.98]	[-29.76,-1.535]	[0.06,0.06]	[0.03,0.03]	[0.00,0.00]	[0.00,0.00]
26	[0.92,0.97]	[-23.83,2.87]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
27	[0.96,0.99]	[-24.38,5.15]	[0.09,0.09]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
28	[0.98,1.01]	[-23.99,6.39]	[0.04,0.04]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
29	[1.00,1.02]	[-23.64,7.19]	[0.17,0.17]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
30	[0.88,0.97]	[-30.51, -2.04]	[0.03,0.03]	[0.010.01]	[0.00,0.00]	[0.00,0.00]
31	[0.85,0.96]	[-31.35, -2.82]	[0.05,0.05]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
32	[0.88,0.98]	[-29.90, -2.78]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
33	[0.88,0.98]	[-29.97, -2.79]	[0.03,0.03]	[0.019,0.01]	[0.00,0.00]	[0.00,0.00]
34	[0.92,0.99]	[-23.67,0.44]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
35	[0.93,1.00]	[-23.31,0.53]	[0.05,0.06]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
36	[0.94,1.00]	[-22.92,0.63]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
37	[0.95,1.01]	[-22.54,0.71]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
38	[0.99,1.03]	[-21.24,1.05]	[0.14,0.14]	[0.07,0.07]	[0.00,0.00]	[0.00,0.00]
39	[0.95,1.01]	[-22.63,0.71]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
40	[0.94,1.00]	[-23.06,0.60]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
41	[0.98,1.01]	[-25.61,0.86]	[0.06,0.06]	[0.02,0.03]	[0.00,0.00]	[0.00,0.00]
42	[0.94,0.99]	[-26.96, -0.21]	[0.07,0.07]	[0.04,0.04]	[0.00,0.00]	[0.00,0.00]
43	[1.00,1.02]	[-22.66,3.13]	[0.02,0.02]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
44	[1.00,1.0388]	[-19.97,0.91]	[0.11,0.12]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
45	[1.03,1.05]	[-16.44,1.19]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
46	[1.04,1.07]	[-18.83,1.79]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
47	[1.01,1.05]	[-19.94,1.79]	[0.00,0.00]	[0.29,0.29]	[0.00,0.00]	[0.00,0.00]
48	[1.00,1.04]	[-20.54,1.49]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
49	[1.01,1.05]	[-22.08,0.97]	[0.17,0.18]	[0.08,0.08]	[0.00,0.00]	[0.00,0.00]
50	[1.00,1.04]	[-23.67,1.13]	[0.20,0.21]	[0.10,0.10]	[0.00,0.00]	[0.00,0.00]
51	[1.03,1.06]	[-24.04,2.43]	[0.17,0.18]	[0.05,0.05]	[0.00,0.00]	[0.00,0.00]
52	[0.96,0.99]	[-25.50,6.08]	[0.04,0.04]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
53	[0.94,0.98]	[-26.25,5.59]	[0.20,0.20]	[0.10,0.10]	[0.00,0.00]	[0.00,0.00]
54	[0.99,1.01]	[-25.55,5.31]	[0.04,0.04]	[0.01,0.01]	[0.00,0.00]	[0.00,0.00]
55	[1.04,1.05]	[-24.40,5.39]	[0.06,0.06]	[0.03,0.01]	[0.00,0.00]	[0.00,0.00]
56	[0.93,0.99]	[-27.03, -0.70]	[0.07,0.07]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]
57	[0.92,0.99]	[-27.46, -1.25]	[0.06,0.06]	[0.02,0.02]	[0.00,0.00]	[0.00,0.00]

load flow method encompass all the solutions obtained from conventional load flow simulations thus guaranteeing the results. The solutions obtained by the proposed method provide more information in qualitative terms as the values obtained also include the unknown uncertainties. Conventional load flow solutions give only deterministic values which are approximated results.

Interval methods are often affected by overestimation, hence the computed error bounds become overly pessimistic. Even though (Vaccaro et. al, 2010) have stated that use of interval arithmetic in power flow analysis leads to solutions which are not useful for practical applications, alternative evaluation schemes can be applied to overcome this problem. The dependency problem and the wrapping effect are particular sources of overestimation in interval computations. Dependency problem occurs due to the failure of interval arithmetic to identify the different occurrences of the same variable. For reducing both the dependency problem and the wrapping effect, interval arithmetic has been extended with symbolic computations using Taylor models in (Neher, Jackson and Nedialkov, 2007). The dependency problem can be also eliminated by applying a suitable extension of the Jacobian matrix as discussed in (Kearfott, 1991). Consequently, this technique can be made computationally more robust and reliable, thus yielding better and accurate solutions.

Load flow analysis helps to ensure that cables, transformers, lines are sized properly to carry the variable load. From the results, it can be determined whether the system voltages remain within specified limits and the equipments are not getting overloaded. The profiles of bus voltages and angles help us to identify real and reactive power flows and minimize the transmission losses. Load flow studies become important in planning and expansion while ensuring that each generator runs within the specified limits and demand be met without overloading the power infrastructure. The reactive power bounds provide us an information regarding the injection of reactive power into the system, in order to keep the power factor close to unity.

Also, in the initial stages of planning and design studies of power systems, the proposed technique will be useful to save time, effort, and the resources required. The results obtained by the proposed method take into account the demand error which is very significant, whence, the load uncertainty is not accounted for, the load flow solutions obtained would be a 'snapshot' for a single specific configuration and operating conditions of the power system. If uneven variations in active and reactive powers in different buses are considered then we can arrive at a more realistic solution. The same is being considered as a part of the future work.

New techniques that are computationally robust and reliable are needed for the analysis of power systems with uncertainties, especially in view of the increasing use of renewable power sources, such as wind, hydro and solar power which are highly variable. However, using interval arithmetic computational requirements are greater than the conventional load flow methods.

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