Probabilistic Modeling of Fatigue Damage in Steel Box-Girder Bridges Subject to Stochastic Vehicle Loads

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Abstract: Structural failures caused by fatigue damage of several famous steel bridges lead to a wide concern to international bridge industry. A probabilistic fatigue damage modeling method was presented and applied to rib-to-deck details of steel box-girder bridges. In order to solve the time-consuming problems of the bridge finite element analysis under traffic flow loads, a response surface method was used to approximate the function between vehicle axle weight and the equivalent fatigue stresses with a few training data. Finally, the fatigue damage model was applied to the reliability assessment of steel box-girder bridges and the influence of traffic flow parameters on structural fatigue reliability was studied. The numerical results indicate that: the higher occupancy rate of heavy vehicles flow in the slow lane is the main reason for the decrease of the fatigue reliability of corresponding rib-to-deck details compared with the fast lane; the increase of the vehicle axle weight causes a rapid decrease of the fatigue reliability index of the steel box girders. When the annual linear growth factor increases from 0 to 1%, the fatigue reliability index of rib-to-deck detail in the slow lane decreases from 3.42 to 0.72. There is a promising application for stochastic fatigue vehicle flow model and the probabilistic model of fatigue damage.

Keywords: steel box girder, stochastic traffic flow, response surface, fatigue reliability, rib-to-deck

1 Introduction

In recent years, structural failures caused by fatigue damage of several famous steel bridges lead to a wide concern to international bridge engineering (Wolchuk, 2011). However, under the continual growth of vehicle load, several existing steel box-girder bridges in China have suffered from fatigue critical problems (Pan et al., 2011). Since the vehicle load is random in nature, the vehicle induced fatigue damage of actual bridges is a stochastic process. It has been proved to be an urgent issue for probability analysis of fatigue damage of steel box-girder bridges subjected to random vehicle loads**.**

For Long-term structural health monitoring system (SHMS), the sensors are expensive and sensitive to environment factors such as temperature. Besides, finite element method and laboratory fatigue test are common methods to acquire the fatigue stress spectrum. Zhang et al. (2011) indicated the influence of vehicle speed and road surface roughness on fatigue damage. Wang et al. (2013) studied fatigue performance of steel bridge and established probability model utilizing nondestructive detection technology and the fracture mechanics method. The typical fatigue truck load model specified in the native design codes are usually used to obtain structural stress spectrum (Castillo et al., 2014). However, the typical fatigue truck models with deterministic parameters are not suitable to conduct probabilistic fatigue analysis.

In fact, random traffic flow load is appropriate for probabilistic analysis. More effort should been done on the application of traffic flow load model on probabilistic modeling of bridge fatigue damage.

In this paper, a fatigue truck load model is developed for probabilistic modeling of highway bridges. In order to solve the time-consuming problems of the bridge finite element analysis under traffic flow loads, a response surface method is used to approximate the function between vehicle axle weight and the equivalent fatigue stresses. Finally, the fatigue damage model is applied to the reliability assessment of steel box-girder bridges and the influence of traffic flow parameters on structural fatigue reliability is studied.

2 Stochastic Vehicle Flow Model

Due to the different national conditions, guidelines for Design and Maintain of Orthotropic Steel Deck in China recommends a four-axle fatigue vehicle with axle load of 50kN, 100kN, 90kN and 90kN. It is available to obtain fatigue damage values of structures via typical fatigue vehicle with deterministic load mentioned above, but it is not applicable for probability analysis.

In general, the effective stress influence line for the orthotropic steel bridge deck is confined to the region between two diaphragm plats (which is 3.2m in this case study). (Deng et al., 2014). Thus, the parameters in the stochastic vehicle load have different influence on structural fatigue damage. It is acknowledged that vehicle weight, vehicle type, and driving lane have a greater influence on fatigue damage compared with vehicle gap and vehicle speed. Thus, it is appropriate to set the weight, vehicle configuration and driving lane as random variables. Therefore, the randomness of vehicle gap and driving speed are ignored. A framework illustrating the procedures of simulating the traffic flow is shown in Figure 1.

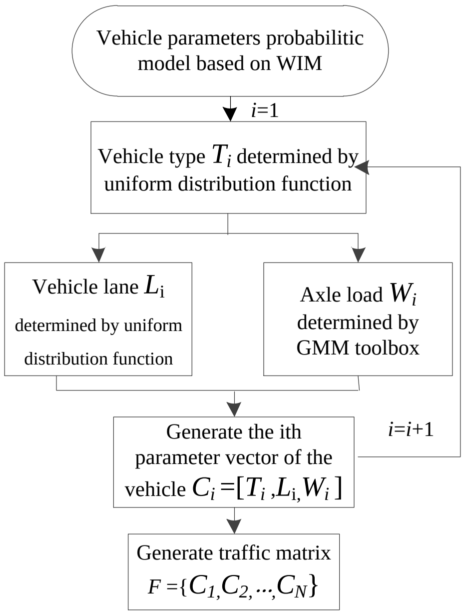

Figure 1. A framework for stochastic traffic flow simulation based on the Monte-Carlo approach (In this article, GMM refers to Gaussian mixture model).

In Figure 1, probability model of vehicle was derived from data obtained by weigh-in-motion (WIM). In Matlab, uniform distribution and GMM functions are used to generate the truck sample. Vehicle matrix C_i was generated by three parameters, including driving lane L_i, vehicle weight W_i, and vehicle configuration T_i. Finally, vehicle matrix F can be obtained in time domain.

The statistic of vehicle of a highway bridge in China is shown in Table I. Note that vehicle with a less vehicle weight than 30 kN were ignored. The axle load of AW_{62} fitted by Gaussian mixture model (GMM) is shown in Figure 2. With these probabilistic models, the stochastic vehicle flow are simulated and shown in Figure 3.

Table I. Statistics of vehicles of a highway based on the WIM system.

Vehicle type	Description	Occupancy rate /%		
		Vehicle type	Slow lane	Fast lane
V_1	Light car	34.64	36.64	63.36
V_2	Truck with 2 axles	26.12	84.58	15.42
V_3	Truck with 3 axles	8.58	91.08	8.92
V_4	Truck with 4 axles	10.24	96.42	3.58
V_5	Truck with 5 axles	4.93	92.60	7.40
V_6	Truck with 6 axles	15.49	98.08	1.92

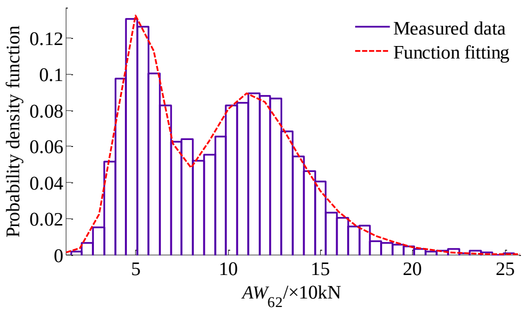

Figure 2. PDF of the axle weight AW62.

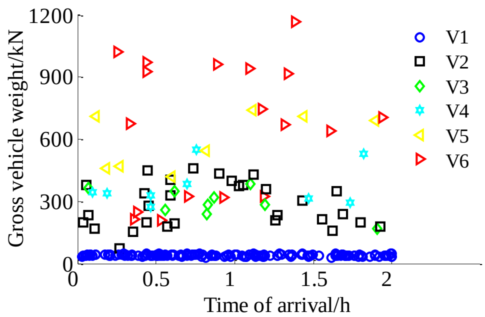

Figure 3. A stochastic fatigue traffic flow sample.

The establishment of stochastic vehicle flow model provides a basis for probability analysis of fatigue damage.

3. Proposed Computational Framework

Response Surface Method (RSM) is proposed to approximate the response function between vehicle parameters and structure fatigue stress. The advantage of the RSM approach is the efficient of computational effort because only small samples are involved in the computation. GMM model with multiple parameters is used to approximate the PDF of fatigue damage. The computational framework is summarized in Figure 4.

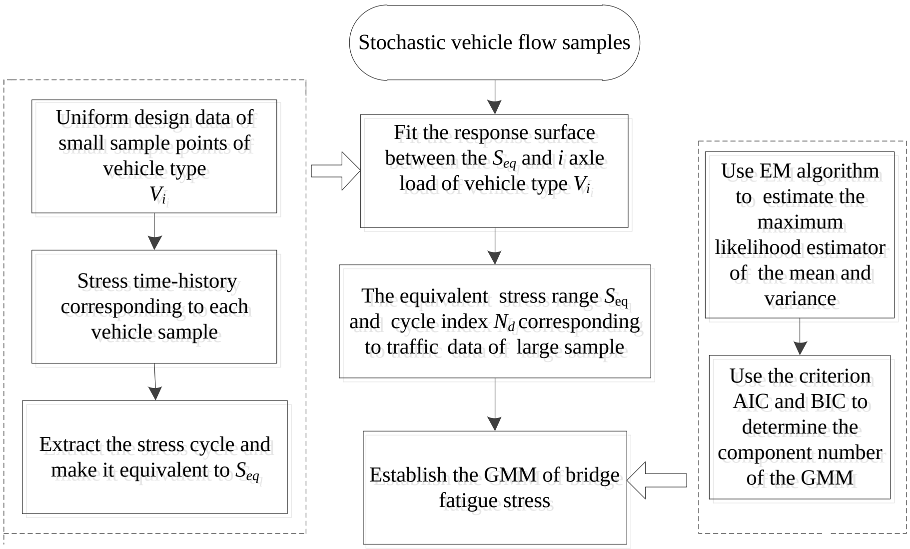

Figure 4. Probability analysis process of fatigue stress.

AW_{in} refers to the nth axle load of the V_ith vehicle type, EM represents the maximum expected value, AIC and BIC are Akaike information criterion and Bayesian criterion in information statistics, respectively. The crucial step in the flow chart is approximating the response surface between axle load and S_{eq}.

Structural response function is the kernel content for the RSM approach (Kang et al., 2010). In general, response surface method is used for approximate nonlinear function associate with small samples. Taking symbol of Vi for example, more than 2i+1 sample points is necessary to determine expression of response surface method. The general formulation of RSM approach is (Kang et al., 2010).

$$Z = G(X) \approx a + \sum_{j=1}^{i} b_j \cdot X_j + \sum_{j=1}^{i} c_j \cdot X_j^2 \tag{1}$$

where, X is a random variable, which is axle load in this paper, a, b, c are undetermined coefficients. It's worth noting that establishment of response surface function is available only with 2i+1 times of finite element calculation.

Once the time-history of stress ranges are obtained for the FE model, the optimal probability model of GMM for fatigue stress can be established based on Criterion BIC and AIC. The criterion BIC and AIC are available in Deng et al. (2014).

4. Probabilistic Analysis of Fatigue Damage for Steel Box Bridge

Fatigue damage of steel bridge, under vehicle loading, is manifested in the location of rib-to-deck(Chen et al., 2014). The rib-to-deck details is specified as number 71 in Eurocode3, the corresponding strength coefficient proves respectively to be K_C=7.16×10^{11}, K_D=1.90×10^{15}. The following formulas are available on the basis of linear damage rule and *S-N* curve, with the introduction of equivalent stress-range S_{eq} (Guo and Chen, 2011).

$$D_{eq} = \frac{N_d S_{eq}^5}{K_D} = \sum_{S_i \geq \sigma_D} \frac{n_i S_i^3}{K_C} + \sum_{S_j < \sigma_D} \frac{n_j S_j^5}{K_D} \tag{2a}$$

$$S_{eq} = \left[\frac{\sum_{S_i \geq \Delta\sigma_D} \frac{n_i S_i^3}{K_C} + \sum_{S_i < \Delta\sigma_D} \frac{n_j S_j^5}{K_D}}{N / K_D} \right]^{1/5} \tag{2b}$$

$$N = \sum_{S_i \geq \Delta\sigma_D} n_i + \sum_{S_i < \Delta\sigma_D} n_j \tag{2c}$$

where n_i is the number of stress cycles, for $S_i \geq \Delta\sigma_D$. Otherwise it turns into n_j. D_{eq} is equivalent cumulative damage. Fatigue failure of the steel decks, mainly includes weld toe cracks and welding cracks. The finite element model of a standard girder is shown in Figure 5.

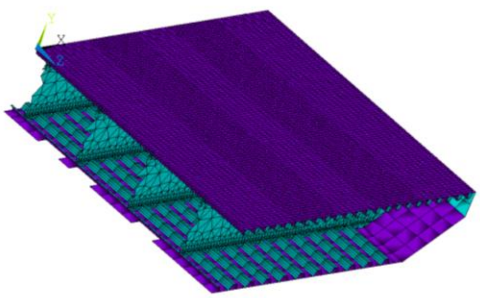

Figure 5. Finite element model of a steel box girder.

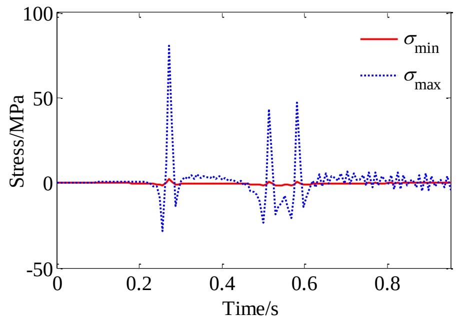

Figure 6. Stress-time curves of roof U-rib of the steel box girder.

In fact, the road surface has a certain effect on the fatigue stress. The loading area for the tires is revised as 3.4cm×33.4cm for the front wheels, and 73.4cm×33.4cm for the rear wheels. Stress-time history for the rib-to-deck details is calculated in FE model and shown in Figure 6. The vehicle driving speed is 20m/s. In Figure 6, σ_{min} and σ_{max} are the minimum and maximum stress time-histories. This stress time-history in Figure 6 obviously contains three peak values. They are corresponding to the location crossed by three vehicle axles. S_{eq} could be obtained through disposing of aforementioned time-histories in accordance with rainflow counting method and Eq.(4). The response surface was approximated with S_{eq} and AW_{31}, AW_{32} with Eq.(1). The simulated response function is shown in Figure 7.

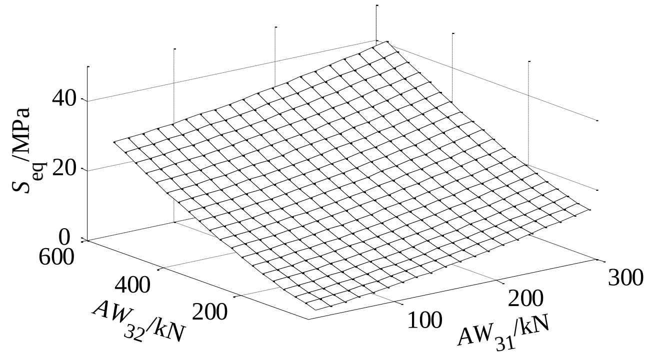

Figure 7. Response surface of Seq and axle weigh of V3.

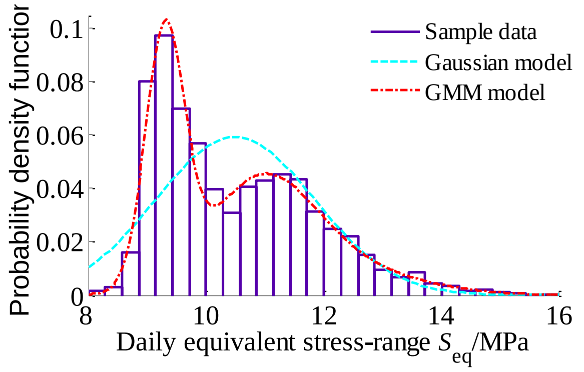

Figure 8. Fatigue stress probability density of the rib-to-deck detail of driveway under the random traffic flow samples.

Equivalent stress-range S_{eq} of rib-to-deck for steel box girder, under the action of traffic samples, can be obtained with vehicle type corresponding to each response surface function. According to response surface algorithm, vehicles with three axles corresponds to seven sample points, while vehicles with six axles corresponds to thirteen sample points. Thus, the whole vehicle types correspond to fifty sample points. Using vehicle type V3 as an example, in the case, calculation time of the condition of one hundred thousand truck flow within 100 days requires 49 days, compared with 36 minutes with application of response surface method mentioned above. It can be verified that the introduction of response surface method can solve the time-consuming problem in fatigue stress analysis.

In accordance with model establishment method of GMM shown in Figure 2, probability density curve of equivalent stress-range can be available shown in Figure 8. GMM parameters are illustrated in Table II, where a_i refers to the weight of Gaussian mixture distribution, μ_i refers to the mean value of the *i*th Gaussian distribution function, σ_i is the standard deviation of the *i*th Gaussian distribution function.

Table II. Parameters of GMM of S_{eq}.

a_1	0.38
a_2	0.62
μ_1	9.43
μ_2	3.74
σ_1	3.44
σ_2	2.31

In Figure 8, the probability density function of equivalent stress-range has approximately two peaks, The two peaks are corresponding to probability density function of axle load in Figure 2. They are associated with empty and full loading condition. With the probability density functions of S_{eq}, GMM is better to approximate the PDF of fatigue stress compared with a single Normal distribution. In Table II, the two mean parameters of GMM, is associated with the two peak values of probability density in Figure 8.

The probability model of the fatigue damage indicated the feasibility to analyze fatigue stress of steel box girder bridges under stochastic vehicle load.

5. Application of the Probabilistic Model: Reliability Assessment

One of application for fatigue damage model under the action of stochastic vehicle flow is fatigue reliability assessment. Limit state function with vehicle flow parameters taken into consideration could be developed as follows, on the base of linear damage rule.

$$g_n(\boldsymbol{X}) = \Delta - D_n(\mathrm{X}) = \Delta - 365 S_{\mathrm{eq}}^5 \cdot N_{\mathrm{d}} \cdot w \cdot \sum_{i=1}^{n} \left[1 + (i-1)a\right]^5 / K_{\mathrm{D}} \tag{3a}$$

$$N_{\mathrm{d}} = ADTT_{\mathrm{sl}} \cdot \sum_{i=1}^{n} p_i \cdot n_i \tag{3b}$$

where n (unit in year) denotes the service time. w is transverse distribution coefficient of wheelmark. a is annual increase rate of axil weight. $\varDelta$ stands for critical parameters for fatigue damage. p_i and n_{i} represent respectively occupancy rate and numbers of axletree of vehicle type i. $ADTT_{\mathrm{sl}}$ is daily traffic volume. The statistical data of random variable X, associated with four parameters, is illustrated in Table III.

Table III. Stochastic of random variables.

Variable/Unit	Distribution type	The mean	The standard deviation
S_{deq}/MPa	GMM	As shown in Table II	
N_{d}	Normal distribution	Eq(2)	Eq(2) ×0.1
$\varDelta$	Lognormal distribution	1	0.3
K_{D}	Lognormal distribution	3.47×10^{14}	1.56×10^{14}
w	Normal distribution	0.78	0.078

In allusion to explicit functions shown in Eq.(3), reliability index, calculated by traditional first order second moment method, will encounter bigger errors, due to higher nonlinear frequency and various distribution types of random variables. In this paper, random samples obeying normal distribution, lognormal distribution and Gaussian mixture distribution functions were generated with the integration of Monte Carlo sampling method and MATLAB software.

Firstly, variation tendency concerning fatigue reliability index and failure probability of rib-to-deck, at the location of slow and fast lane, has been analyzed in Figure 9, without the consideration of increase in axle load.

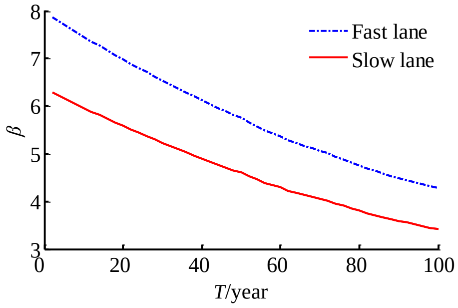

Figure 9. Fatigue reliability index of the rib-to-deck detail during the operation period.

As the Figure 9 illustrated, fatigue reliability index at rib-to-deck on slow lane drops from 6.29 to 3.42. With the growth of service time, although a downward trend of fatigue reliability index occurs both at rib-to-deck on slow and fast lane, reliability index of slow lane is relatively lower than that of fast lane. This occurs because more than 90% of trucks with at least 3 axis act generally on slow lane shown in Table 1.

Due to the exponential relationship between axle load and fatigue damage, growth of the axle load will have great influence on fatigue reliability. With the annual growth factor of axle load—a set from 0 to 1%，the variation tendency at rib-to-deck on slow lane is shown in Figure 10.

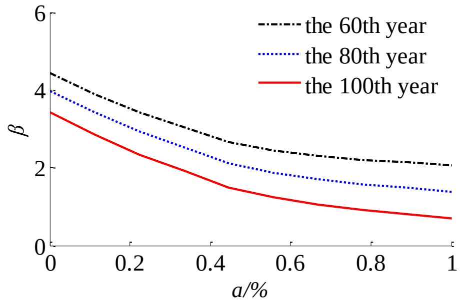

Figure 10. Influence of increase of vehicle weight on fatigue reliability index.

As shown in Figure 10, fatigue reliability index at rib-to-deck of steel box girder drops rapidly under the consideration of growth of vehicle weight. With the annual growth coefficient of vehicle weight increasing from 0 to 1%, the corresponding fatigue reliability indexes in the 60th, 80th, 100th year are found to change from 4.43 to 2.01, 3.98 to 1.30 and 3.42 to 0.72 in the meantime. It can be found that fatigue reliability index drops along with the growth of the bridge service time in this regard. In conclusion, the service time and increase of axle load have significant influences on fatigue reliability of welded details.

In the operation management of the bridges, particular attentions should be given to statistical analysis about operating vehicles. It is available to take certain measures to controlled traffic, based on the reliability assessment method described in this paper, when statistical parameters of vehicle grow excessively fast.

6. Conclusion

Fatigue truck models with deterministic parameters were developed to be a stochastic vehicle flow model. In order to solve the time-consuming problems of the bridge finite element analysis under traffic flow loads, a response surface method has been used to approximate the function between vehicle axle weight and the equivalent fatigue stresses with a few training data. A probabilistic fatigue damage modeling method was presented and applied to rib-to-deck details of steel box-girder bridges. Finally, the fatigue damage model was applied to the reliability assessment of steel box-girder bridges. Research results provide theoretical foundation for operation management and traffic restrictions of bridges. Main conclusions are summarized as follows.

(1) Stochastic vehicle flow model involving statistical characteristics of vehicles has a certain application prospect in probability analysis of fatigue damage for steel bridge and reliability assessment.

(2) Higher occupancy of heavy vehicle on slow lane has been proved to be significant cause, which leads to fatigue reliability index of slow lane much lower than that of fast lane.

(3) The growth of axle load results in rapid decline in fatigue reliability index of steel box girder. As the annual growth factor of vehicle weight increase from 0 to 1%, fatigue reliability index of the 100th year drops from 3.42 to 0.72.

Still at issue is influence which vehicle speed and road surface roughness, in stochastic vehicle flow model, make on fatigue damage. Growth factors of traffic volume and vehicle weight remain to be integrated by long-term monitoring data.

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