

Dynamic Reliability Assessment for Long-Span Bridges under Heavy Stochastic Traffic Flows

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Abstract: Long-span highway bridges simultaneously suffer from a large number of vehicle loads. With the ever-increase of the traffic loads, the safety problem of the in-service bridges is particularly serious. A framework for analysis the dynamic reliability of long-span bridges under stochastic traffic flows was proposed. The framework integrates the monitored traffic data, vehicle-bridge interaction theory, and dynamic reliability theory. A suspension bridge with main-span of 820m was chosen as an engineering prototype for the purpose of illustrating the feasibility of the proposed framework. The numerical results indicate that: the stochastic traffic flow model is appropriate to be applied in reliability analysis of bridges, because the probabilistic information was included in the stochastic traffic flow. The corresponding probabilistic model of dynamic response of bridges can be obtained by the numerical simulation of vehicle-bridge interaction. The busy traffic flow was demonstrated to be the main reason for the displacement first passage failure of the suspension bridges. Furthermore, the failure probability increases with the growth of the occupancy rate of the busy traffic flow in the daily traffic flow.

Keywords: dynamic reliability, stochastic traffic flow, suspension bridge, first-passage, root-mean-square

1. Introduction

In recent years, a steady increase in the vehicle volume and cross vehicle weight, which has been caused by the rapid growth and expansion of urban developments and subsequent increase in inter-city and interstate transportation, have caused a threat to the safety of bridges (Chen and Wu, 2010). The overloading traffic due to this vehicle volume increase has become the main factor resulting in shortening the service life and even causing collapse of bridges in most countries (Deng et al., 2015). With the widespread increase of vehicle volume and cross vehicle weight, several in-service bridges built decades ago, utilizing the old design codes, are damaged or even collapsed. Therefore, the structural safety of existing bridges caused by sustainable growth of traffic volume and assessing the load-carrying capability are serious issues that should be carefully studied.

The commonly used method for the evaluation of the vehicle load in bridge design and safety assessment are specifications in the national design codes developed in the individual countries. The practical application of these codes have proved to be successful as they allowed for keeping in operation many bridges that no longer met the design criteria but were otherwise able to safely carry the applied loads. Because of the success of these national codes and more recent research findings the possibility of developing a new Eurocode for bridge safety assessment is under consideration. In the design specifications, the vehicle loads were calculated based on the statistical data of collected from large volume of vehicles in multiple regions. Due to changes that have occurred in the traffic flow and volume in recent

years, these data may no longer be suitable or relevant for all existing bridges. Some the shortcomings are for instance the design loads considered in these codes which do not take into consideration the random nature of the actual vehicle flow and vehicle loads (Bu et al., 2006). There are additional shortcomings in the existing codes. For instance, the design load considered in these codes are based on an overall estimation of the number of vehicles. Thus, although slender long-span bridges may carry a large number of heavy trucks simultaneously, this factor has not been considered in these code. Traffic flow is a more realistic model to simulated vehicles passing over lender long-span bridges instead of vehicle load that is based on estimating the number of vehicles (Han et al., 2015). Subsequently, a more realistic vehicle model with a focus on estimating traffic flow should be developed and incorporated. Stochastic traffic flow is the most effective and realistic model that needs to be considered especially for long-span bridges since the random loading caused due to traffic flow results in direct and severe vibration of bridges, compared with, and in contrast to, the transient vibration of a single vehicle. There are uncertainties associated with the vehicle physical parameters as well as its dynamic characteristics. Xiang et al. (2007) evaluated the bridge structural reliability considering the vehicle-bridge dynamic interaction, however, the reliability evaluation method that they utilized is a traditional static reliability method which does not take into account the random vibration of the bridge caused by the road surface roughness and the bridge-vehicle interaction.

This paper aims at developing a general framework to evaluate the reliability of long-span bridges subjected to stochastic traffic flows utilizing the first-passage approach formulated on the basis of Rice's formula. As the first step, three types of stochastic traffic flow, including free flow, moderate flow, and busy flow, are simulated adopting the cellular automation approach and weigh-in-motion data. Under the long-time effect of simulated traffic flows, the stochastic characters of the dynamic behavior of bridges are simulated and computed utilizing a simplified traffic-bridge interaction theory. Finally, a suspension bridge with a mid-span of 820m is selected to conduct a numerical study, which includes incorporating a large number of traffic parametric studies, such as traffic density and service time.

2. Theoretical basics

2.1 VEHICLE-BRIDGE INTERACTION

The vehicle model and bridge model are established separately and combined with their coupling equations. For the vehicle models, the transitional method is adopting several mass blocks, springs, and damping dashpots. The full vehicle model, half vehicle model, and quarter vehicle model were defined by many researchers (e.g. Yin et al., 2012). In order to simplify the illustration in the present paper, a half vehicle model is shown in Figure 1.

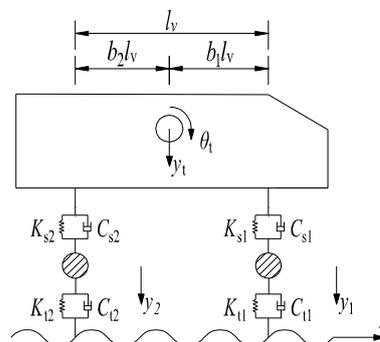


Figure 1. A half vehicle model in the study.

More details for the half vehicle model as well as the relevant information can be founded by Yin et al. (2012). The general coupled equations of the vehicle-bridge system can be constructed through a relatively lengthy derivation as

$$\begin{bmatrix} M_v & 0 \\ 0 & M_b \end{bmatrix} \begin{Bmatrix} \ddot{u}_v \\ \ddot{u}_b \end{Bmatrix} + \begin{bmatrix} C_v & 0 \\ 0 & C_b \end{bmatrix} \begin{Bmatrix} \dot{u}_v \\ \dot{u}_b \end{Bmatrix} + \begin{bmatrix} K_v & 0 \\ 0 & K_b \end{bmatrix} \begin{Bmatrix} u_v \\ u_b \end{Bmatrix} = \begin{Bmatrix} F_{vg} + F_{vb} \\ F_{bg} + F_{bv} \end{Bmatrix} \quad (1)$$

where, M_v , C_v , and K_v are the mass, damping, and stiffness matrices of the vehicle, respectively; M_b , C_b , and K_b are the mass, damping, and stiffness matrices of the bridge, respectively; u_v and u_b are the displacement of the vehicle and the bridge, respectively; F_{vg} and F_{bg} are the self-weight of the vehicle and the bridge, respectively; F_{vb} and F_{bv} are the vehicle-bridge interaction forces.

Currently, many studies on the multiple-vehicle effects on bridges suggest that the interaction effects between different vehicles on the same long-span bridges, such as cable-stayed bridges and suspension bridges, are insignificant. Furthermore, the dynamic interactions between different vehicles may become negligible. Therefore, individual vehicle may be approximated as a directly applied force on the bridge, independent of other vehicles. An equivalent dynamic wheel loading (EDWL) approach was presented by Chen and Wu (2007). Replacing the physical moving vehicles on the bridge by EDWL, the fully coupled equations in Eq. (1) can be simplified to

$$M_b \ddot{u}_b + C_b \dot{u}_b + K_b u_b = F_{bg} + F_{eq}^{wheel} \quad (2)$$

$$\{F(t)\}_{eq}^{wheel} = \sum_{j=1}^{n_v} \left\{ [1 - R_j(t)] G_j \cdot \sum_{k=1}^{n_d} \{ h_k [x_j(t) + \alpha_k [x_j(t) d_j(t)]] \} \right\} \quad (3)$$

where, R_j , G_j , x_j , and d_j are the dynamic wheel ratio, self-weight of the j th vehicle, longitudinal location, and transverse location of the gravity center of the j th vehicle on the bridge, respectively; h_k and a_k are the k th vertical and the torsional mode shapes of the bridge. The total number of vehicles n_v changes with time depending on the simulation results of the stochastic traffic flow. The feasibility analysis has been conducted by Chen and Cai (2007) by comparing the bridge response estimations using fully coupled interaction analysis and the EDWL approach.

Due to the random nature of the traffic loads, the bridge responses under stochastic traffic flow are random processes (Wu and Law, 2011). Consequently, a convergence analysis is required for the purpose of obtaining rational estimations of the probabilistic dynamic behavior of the bridge response. In addition, the displacement of long-span bridges highly depend on the vehicle density (Chen and Wu, 2010). Even through the vehicle density is time variant in one day, the interval vehicle density exhibits stationarity in a time domain. On this basis, the traffic model can be divided into several models in the time domain according to the statistics of the traffic density. Once the traffic model is determined, the sample of the responses will exhibit ergodicity on the premise of a long analytical time.

2.3 FIRST-PASSAGE FORMULATIONS

The first-passage probability, which describes the probability that a scalar process exceeds a prescribed threshold during an interval of time, is of great engineering interest. The probability is essential for estimating the serviceability reliability of bridges. The first-passage probability of a zero-mean stochastic process $X(t)$ over a prescribed double-sided threshold $|x|=a$ during an interval time $t \in (0, \tau)$ can be written as

$$p(a, \tau) = P(a \leq \max_{0 \leq t \leq \tau} |X(t)|) \quad (4)$$

In general, there is no exact solution for the probability. Rice (1945) adopted Poisson assumption to approximate the probability of Eq. (4) and presented the well-known Rice's formula. For the random vibration of bridges under traffic loads, the mean value of responses of bridge beams are greater than zero in theory, while they are far less than threshold a for a safety design. In this case, the crossing rate under the threshold $-a$ can then be written as

$$v^-(-a) = \frac{\sigma_{\dot{X}}}{2\pi\sigma_X} \exp\left(-\frac{(a - E_X)^2}{2\sigma_X^2}\right) \quad (5)$$

where, E_X is the mean value of random process X , \dot{X} is the derivation of X , σ_X and $\sigma_{\dot{X}}$ are the root mean square of X and \dot{X} , respectively. Note that the Rice formula adopts two assumptions including the over-crossing theory and the Poisson distribution. Thus, the validity for applying to this study should be further investigated by comparing to the Monte Carlo simulation. In the present study, the results calculated by the Rice formula can be considered as references.

Considering the density of stochastic traffic flow, the first-passage probability of bridge girders under multiple types of traffic flows can be written as

$$p(a, T) \cong 1 - \sum_{i=1}^n \exp[-v_i^-(-a) \cdot \rho_i \cdot T] \quad (6)$$

$$\rho_i = \frac{t_i}{T} = \frac{t_i}{\sum_{i=1}^n t_i} \quad (7)$$

$$v^-(-a, t_i) = \frac{\sigma_{\dot{X}(i)}}{2\pi\sigma_{X(i)}} \exp\left(-\frac{(a - E_{X(i)})^2}{2\sigma_{X(i)}^2}\right) \quad (8)$$

where, the entire time T is divided into n intervals indicated as t_i according to the density of stochastic traffic flow, such as free traffic flow, moderate traffic flow, and busy traffic flow; v_i and ρ_i correspond to the crossing rate and occupancy of the i th traffic term, respectively; $X(i)$ and $\dot{X}(i)$ denote the structural response and the corresponding time derivative under the i th traffic term, respectively; and $\sigma_{X(i)}$ and $\sigma_{\dot{X}(i)}$ denote the RMS of the $X(i)$ and $\dot{X}(i)$, respectively.

3. Case study

3.1 WEIGH-IN-MOTION SYSTEM

A weigh-in-motion (WIM) system utilizes scales or pressure sensors embedded into the road pavement to measure parameters of crossing vehicles such as axle weight and speed. These systems are widely used in highways for the purpose of controlling heavy traffic load such as overloaded trucks. Statistical approaches are adopted to establish the probability model of the axle weight, speed, vehicle type, and/or other relevant information with these vehicle data. In general, both the vehicle type and vehicle speed fit normal distribution, the axle weight fits a bimodal distribution, and the vehicle gap, i.e., the gap between two vehicles in a lane, fits a Gamma or Weibull distribution depending on the vehicle density. The bridge loadings models are closely related to vehicle gross weights, axle weight and axle spacing (Miao and Chen, 2002). Although thousands of vehicles cross a the bridge every day and the vehicle types, gross vehicle weights and axle weights are quite different, statistical characteristic can be obtained via the big data of these parameters. If the mathematical distributions of these parameters can be obtained accurately,

bridge live load models can then be easily formulated. A WIM system used in this paper is shown in Figure 2. A diagram illustrating the relationship between the vehicle and the WIM is shown in Figure 3.



Figure 2. A WIM system.

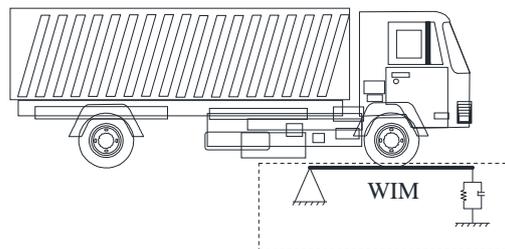


Figure 3. A truck passing on a WIM system.

This system has been working about 5 years and more than ten million vehicle data has been collected. According to the axle configurations, all vehicles collected from the WIM data can be classified into 6 categories. Taking vehicle type 6 (with 6 axles) as an example, its corresponding probability models for axle weight are shown in Figure 4, where the axle weight of the vehicle type 6 AV_{64} follows a Gaussian mixture distribution. The simulated stochastic traffic flow is shown in Figure 5.

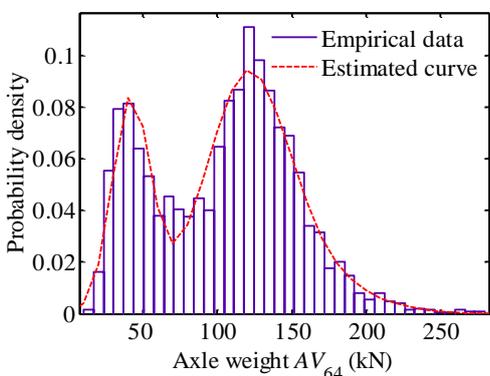


Figure 4. Probability density of parameters in V6.

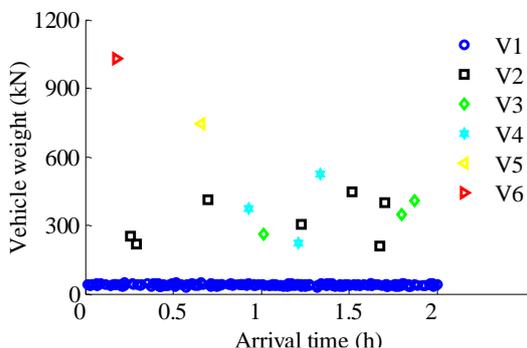


Figure 5. Simulated stochastic traffic flows.

The vehicle density strongly depends on vehicle gaps that changes greatly during the day and night, but it follows the people's daily life style, which means people usually go out in the morning and return in the evening. Thus, in order to establish the probability model of the vehicle gaps, the traffic flow is divided into three parts: free traffic, moderate traffic, and busy traffic, according to the vehicle volume per hour. With the WIM data of three typical dates, the boundary line between busy traffic, moderate traffic, and free traffic is determined as 400 vehicles per hour and 200 vehicles per hour in a driving direction. It can be found that the time of busy traffic flow is about 4 hours (occupancy rate $\rho_1=17\%$), the time of moderate traffic flow is about 6 hours (occupancy rate $\rho_2=25\%$), the time of free traffic flow is about 14 hours (occupancy rate, $\rho_3=58\%$).

3.2 A SUSPENSION BRIDGE

A long-span highway bridge in Sichuan province of China named Nanxi Yangzi River Bridge is shown in Figure 6. A WIM system was installed on the suspension bridge.

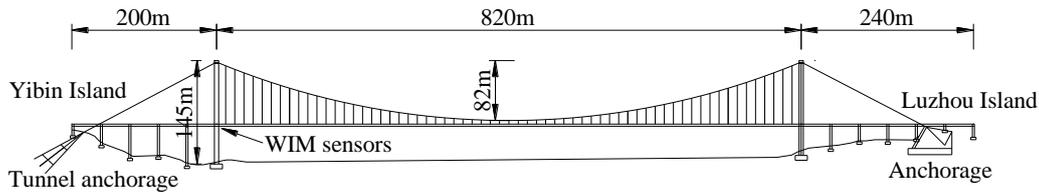


Figure 6. Elevation layout of the prototype bridge.

In order to perform the dynamic response analysis of the prototype bridge under traffic flow, a finite element model was established using ANSYS software. The model is shown in Figure 9, where the Beam44 element was used to simulate the steel box girders and towers, and the Link10 element was used to simulate the cables and the hangers. The roughness displacement was derived after the adoption of roughness factor of $20 \times 10^{-6} \text{m}^3/\text{cycle}$ for good road condition as shown in Figure 7.

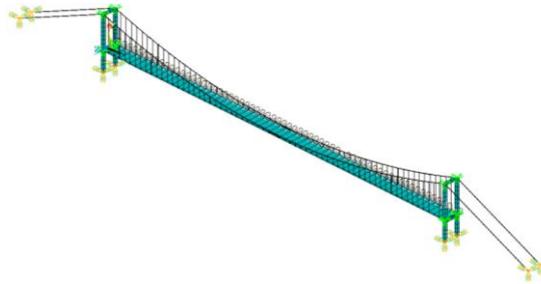


Figure 7. Finite Element Model of Nanxi Yangtze River Bridge.

3.3 RESULTS AND DISCUSSIONS

With the combined use of EDWL approach and the transient analysis function of ANSYS software, the time-history of the suspension bridge girders were obtained and shown in Figure 8. From Figure 8 it can be found that the $L/4$ girders vibrate seriously compared with $L/2$ girders. Furthermore, the displacement of a certain girder under busy flow is larger than that under free flow, which means that the vehicle density has significant impact on the displacement of the girders. The mean value of the displacement under busy flow is larger than that of the moderate flow and free flow.

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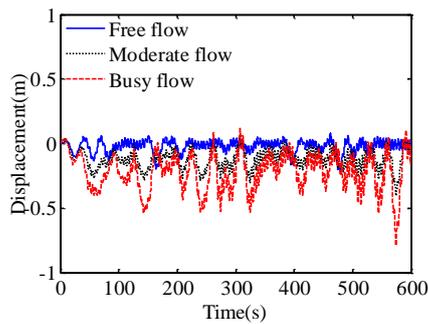


Figure 8. Vertical displacement of the girder in quarter-span under stochastic traffic flow.

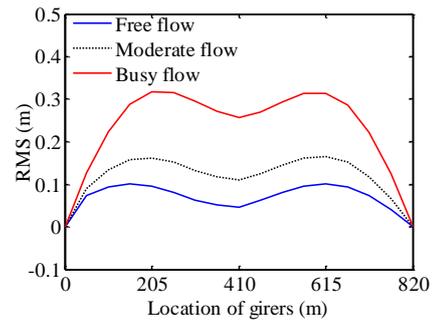


Figure 9. Statistics of displacement of girders under simulated traffic flows.

By the same way, the RMS of all girders can be calculated and the results versus the location of the girders are shown in Figure 9. As can be seen from Figure 9, the following results can be obtained: (1) the RMS of all girders are symmetric and increase rapidly when the free traffic flow changes to be busy traffic flow; (2) the shape of the two type of values are obviously different, where the mean value fits a V-type distribution, while the RMS fits a M-type distribution; and (3) the peak-value of mean value of all girders is located at the mid-span location, while the peak-value of RMS is located at the quarter-span location. A physical interpretation of these results can be explained by the structural modal shape. It is observed that the shape of the RMS is similar to the 1st vertical modal shape, while the shape of the mean value is similar to the 2st vertical modal shape. It was also observed from the animation of the transient analysis that the girder in the quarter-span had a sever vibration compared with the girder in mid-span.

Once the statistical information are obtained from the traffic-bridge dynamic analysis, the first-passage reliability can be calculated by employing the Rice formula. According to the design code in China (MOCAT 2004), the threshold in Eq. (6) is equal to $L/400=2.05\text{m}$, where L is the length of mid-span of the bridge. The first-passage reliability indexes of the quarter-span of the suspension bridge girders subjected to free traffic, moderate flow, and heavy flow are shown in Figure 10.

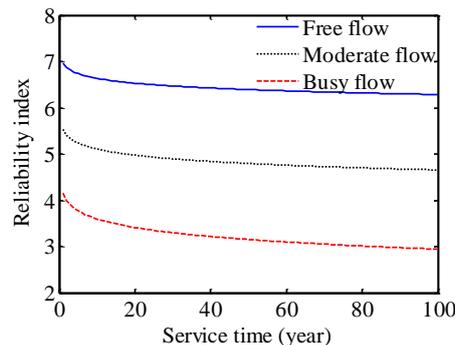


Figure 10. First-passage reliability of the suspension bridge girders subjected to three types of traffic flow.

It can be found that there is an obvious demarcation line between them, where the reliability index for the case of busy traffic flow is the smallest and that of the heavy traffic flow is the greatest. When the service time of the bridge increase from 1a to 100a, the corresponding first-passage reliability index of the

bridge decrease from 4.15 to 2.94. For the same service time, 100a, the reliability of the square-span girder under free flow, moderate flow, and busy flow are 6.28, 4.65, and 2.94, respectively. It can be found that the type of traffic flow, rather than the service time, has a significant influence on the first-passage reliability index of the bridge.

4. Conclusions

A general framework was developed for evaluating the serviceability reliability of long-span bridges subjected to stochastic traffic flow considering the actual stochastic traffic information. Three types of stochastic traffic flows, including busy flow, moderate flow, and free flow, was simulated by the cellular automation approach using the weigh-in-motion data. The traffic flow was transformed into time-variant vertical forces using an equivalent dynamic wheel load approach for the purpose of simulating time-history responses of a bridge girder in the finite element model. Statistical information of the bridge responses, such as mean value and root-mean-square, was then calculated. Then, the first-passage reliability of a case study bridge was calculated based on the established first-passage model utilizing the basis of Rice's formula.

In the case study, three types of traffic flow as well as the corresponding responses of the suspension bridge girders were simulated and subsequently the statistical analysis of first-passage problem was conducted. Numerical results indicated that: (1) under stochastic traffic flow, the shape of distribution for the mean value and RMS of all girders are different, where the mean value fits a V-type distribution; (2) the peak-value of mean value of all girders is located at the mid-span location, while the peak-value of RMS is located at the quarter-span location; (3) considering the design service life (1a to 100a) and the assumed occupancy of statistical busy flow, the first-passage reliability index of the above locations dropped from 4.72 to 3.68. Consideration of busy traffic flow is obvious important for assessing safety of long-span bridges. These numerical results are valuable contributions to the literature.

The result can be checked by the rough Monte Carlo simulation in theory. However, it's an extremely time-consuming problem for carrying out a long-time simulation with a normal personal computer.

Acknowledgements

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